PROPERTIES OF REAL NUMBERS

Axioms of equality

Reflexive property: For any real number a, $a = a$
Symmetric property: For any real numbers a and b, if $a = b$, then $b = a$
Transitive property: For any real numbers a, b and c, if $a = b$ and $b = c$, then $a = c$

Axiom of substitution

For any real numbers a and b, if $a = b$, then a and b may be substituted for each other in mathematical expressions as necessary or useful.

Axioms of transformation

Addition: For each real a, b, and nonzero c, if $a = b$, then $a + c = b + c$
Subtraction: For each real a, b, and nonzero c, if $a = b$, then $a - c = b - c$
Multiplication: For each real a, b, and nonzero c, if $a = b$, then $ac = bc$
Division: For each real a, b, and nonzero c, if $a = b$, then $a \div c = b \div c$

In general:

$$\text{If you} \begin{Bmatrix} \text{add} \\ \text{subtract} \\ \text{multiply} \\ \text{divide} \end{Bmatrix} \text{ equal quantities } \begin{Bmatrix} \text{to} \\ \text{from} \\ \text{by} \\ \text{by} \end{Bmatrix} \text{equal quantities, the resultant quantities are equal.}$$

Field properties

Closure of +: For every real number a and b, the sum, $a + b$ is a unique number which is also a real number.
Closure of $\times$: For every real number a and b, the sum, ab is a unique number which is also a real number.
Associativity of +: For all real numbers a, b, c, $a + (b + c) = (a + b) + c$
Associativity of $\times$: For all real numbers a, b, c, $a(bc) = (ab)c$
Commutativity of +: For every real number a and b, $a + b = b + a$
Commutativity of $\times$: For every real number a and b, $ab = ba$
Distributivity of $\times$ over +: For all real numbers a, b, c, $a(b + c) = ab + ac$, AND $ab + ac = a(b + c)$ AND $a(b - c) = ab - ac$ AND $ab - ac = a(b - c)$
Identity element for +: For every real number a, $0 + a = a + 0 = a$
Identity element for $\times$: For every real number a, $1 \times a = a \times 1 = a$
Inverse elements for +: For every real number a, there exists a unique corresponding real number $-a$ such that $a + (-a) = (-a) + a = 0$
Inverse elements for $\times$: For every nonzero real number a, there exists a unique corresponding real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

Special properties

For each real number a, $0 \times a = a \times 0 = 0$
For all real numbers a and b, if $ab = 0$ then $a = 0$ or $b = 0$ (or both)
For each real number a, $-(-a) = a$
Property of the opposite of a sum: For all real numbers a and b, $-(a+b) = (-a)+(-b)$
Multiplicative property of -1: For each real number a, $-1 \times a = a \times (-1) = -a$
Property of opposites in products: For all real numbers a and b,
$(-a)b = -ab$; $a(-b) = -ab$; $(-a)(-b) = ab\backslash$
Property of the reciprocal of a product: For all nonzero real numbers a and b, $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$

Steps for solving linear equations

1. Combine any similar terms in either member of the equation, after performing any distribution.
2. If there are still indicated additions or subtractions in one or both members of the equation, use inverse operations to undo them.
3. If there are any indicated multiplications or divisions in the variable term, use the inverse operation to find the value of the variable.
4. Check by substituting the value of the variable in the given (original) equation to see whether it satisfies that equation.

Miscellaneous shorthand notation frequently appearing in mathematics

$\exists$ There exists (existential quantifier). This symbol guarantees that at least one value exists that will satisfy the given condition.

$\forall$ For all (choices), for every (universal quantifier) This symbol indicates that every value possible will satisfy the given condition.

$\in$ "is an element of" as in $3 \in \{1,3,5\}$
$\ni, :, |$ "such that" (all three of these symbols are used interchangeably, although generally consistently in any one document)

Specific sets of numbers

Z	{integers}; Z^+, Z^- (to indicate the positive or negative integers)
Q	{rational numbers}; sometimes Ra
$\Re$	{real numbers}
C	{complex numbers}

eg. What does this say?
$\forall a \in \Re, \exists$ a unique corresponding number $(-a) \in \Re \ni (-a) + a = a + (-a) = 0$