Prove that the number of arrangements of a set of n objects which contains p identical objects and q identical objects (of another sort) is $\frac{n!}{p!q!}$.

Consider the composition of the set and a strategy for constructing the arrangements. There are p of one type of object, q of another type of object and $n-p-q$ additional objects (all of these distinct from one another).

To form the desired arrangements we could proceed as follows. You have n spaces in which to put one object per space. Take all p of the identical objects, and place them in spaces. Note that there are ${}_nC_p = \frac{n!}{(n-p)!\,p!}$ ways to do this.

Now take all q of the other identical objects and place them in spaces (from among the remaining vacant spaces). Notice that there are ${}_{(n-p)}C_q = \frac{(n-p)!}{(n-p-q)!\,q!}$ ways to do this.

At this point there are $n-p-q$ spaces left to fill and there are also the remaining $n-p-q$ objects to be used (these objects are distinct from one another). Filling the remaining spaces with the distinct remaining objects involves a permutation (because putting different objects in the spaces will produce different arrangement results). So there are ${}_{(n-p-q)}P_{(n-p-q)} = (n-p-q)!$ ways to do this.

Thus the total number of possible arrangements is found by taking the product of the numbers of ways to do each of the three steps that are involved in forming the arrangements:

$$\frac{n!}{(n-p)!\,p!} \cdot \frac{(n-p)!}{(n-p-q)!\,q!} \cdot (n-p-q)! = \frac{n!}{p!q!}$$

Were there more indistinguishable groups of items, it would just be an extension of what we have done. Position each set of indistinguishable objects via combinations and then as a last step use permutations to count the number of ways that the distinguishable objects will fill in the remaining spaces.