STUDY GUIDE FOR CHAPTER 14

Which of the following can we find? Which are uniquely determined? (and what sort of information would we need to be given? And is the task easy or hard? Do we have any clever techniques or tricks?)

1 A line parallel to a given line through a given point not on the line.
2 A line perpendicular to a given line through a given point not on the line.
3 A plane parallel to a given plane through a given point not on the given plane.
4 A plane perpendicular to a given plane through a given point not on the given plane.
5 A plane parallel to a line (or a line parallel to a plane).
6 A plane perpendicular to a line (or a line perpendicular to a plane).
7 The intersection of two lines in space (or the clear determination that they do not intersect)
8 The intersection of a plane and a line (or a way to argue that they do not intersect)
9 The line of intersection between two planes.
10 Planes tangent to a sphere at the ends of a diameter.
11 The centers of the two spheres of a given radius that would be tangent to a given plane at a particular given point.

Something to be aware of: If a vector equation of a construct in space has just ONE parameter, then it describes a line; if the vector equation of a construct in space has TWO parameters, then it describes a plane.

Given: P(1,2,3), Q(-4,1,5) and R(2,-2,1)

Find appropriate parametric equations, vector equations, rectangular equations, or points as requested.

1. line $\overleftrightarrow{PQ}$
2. the intersection of $\overleftrightarrow{PQ}$ with the xy-coordinate plane (xz-coordinate plane, yz-coordinate plane)
3. points on $\overleftrightarrow{PQ}$ $5\sqrt{30}$ units from Q.
4. the line through R parallel to $\overleftrightarrow{PQ}$
5. the plane PQR—vector and rectangular equations. Where does the plane intersect the *x*-axis, the *y*-axis, and the *z*-axis.
6. the plane perpendicular to line $\overleftrightarrow{PQ}$ through R and the point of intersection of $\overleftrightarrow{PQ}$ with this plane.
7. the perpendicular bisecting plane of $\overline{PQ}$
8. the equation of the sphere in which $\overline{PQ}$ is a diameter.
9. the plane tangent to a sphere in which $\overline{PQ}$ is diameter; tangent at P, tangent at Q. How far apart are the two planes? What is the relationship between the two planes?
10. parametric equations for the line through R perpendicular to $\overleftrightarrow{PQ}$ and the distance from R to $\overleftrightarrow{PQ}$.
11. a rectangular equation of the plane through $(5,\ 1,\ 4)$ and parallel to the plane PQR
12. a vector equation of the line through $(5,\ 1,\ 4)$ and perpendicular to PQR. Name the point of intersection of the line and the plane.
13. Find *X* between *P* and *Q* so that *X* is four times as far from *P* as *X* is from *Q*.
14. Find *Y* so *P* is between *Q* and *Y* and *Y* is six times as far from *Q* as *P* is from *Q*.
15. Find the point of intersection of $\vec{r} = (6+3t)\vec{i} + (-2-t)\vec{j} + (4+3t)\vec{k}$ and $3x - 2y + z = 17$

16. Given: $\vec{a} = i - 3j + 2k$, $\vec{b} = 2i + 4j - k$, $\vec{c} = 3i - 2j - 3k$. Determine each of the following:

a. the cosine of the angle between $\vec{c}$ and $\vec{b}$.
b. The sine of the angle between $\vec{a}$ and $\vec{c}$.
c. The projection of $\vec{b}$ on $\vec{a}$.
d. $\vec{a} \cdot \left(\vec{c} \times \vec{b}\right)$
e. $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$
f. $\vec{a} \times \left(\vec{b} + \vec{c}\right)$
g. the area of the triangle formed by the points A, B, and C (endpoints of the vectors a, b, and c respectively)

17. A normal to a plane has direction cosines $\frac{-5}{3\sqrt{5}}, \frac{2}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}$ and $\vec{r} = (3+4t)i + (-1+4t)j + (1+3t)k$ is a line in the plane. Find:

a. the rectangular equation of the plane.
b. The distance from the origin to the plane.
c. The foot of the perpendicular from the origin to the plane.

18. Find the equations of the lines (in two-space) bisecting the angles formed between the lines $x + 3y = 10$ and $3x + y = 14$.
19. Find the equations of two planes each parallel to the plane $x - 2y + 2z = 6$ and 4 units away from it (the plane).
20. Find equation(s) for the set(s) of points (x, y, z) that are equidistant from the planes $2x - y - z + 6 = 0$ and $x + 2y - 7z + 12 = 0$. What are these structures?
21. A sphere with center $(7, 1, -2)$ is tangent to a plane $2x + 3y - z = 5$.

a. What is the radius of the sphere?
b. What is the vector equation of the line through the center and perpendicular to the plane at the point of tangency?
c. What are the coordinates of the point of tangency?
d. What is the point of tangency of the sphere to a parallel plane at the other end of the diameter and what is the rectangular equation of that tangent plane?

22. Find an equation for the plane containing the point $(5, 1, -3)$ and perpendicular to each of the following planes: $5x - 3y + 2z = 9$ and $3x + y - 4z = 17$.
23. Find the volume of the tetrahedron formed with the following vertices: $(1, 5, -2), (3, 1, 4), (0, -1, 1), (2, -2, 5)$. NB $V = \frac{Bh}{3}$ where B is the area of the base and h is the height.