CHAPTER 15 PRACTICE PROBLEMS

1. Given: $\vec{a} = i - 3j + 2k$, $\vec{b} = 2i + 4j - k$, $\vec{c} = 3i - 2j - 3k$. Determine each of the following:
   a. the cosine of the angle between $\vec{c}$ and $\vec{b}$.
   b. The sine of the angle between $\vec{a}$ and $\vec{c}$.
   c. The projection of $\vec{b}$ on $\vec{a}$.
   d. $\vec{a} \cdot (\vec{c} \times \vec{b})$
   e. $\vec{c} \times (\vec{a} \times \vec{b})$
   f. $\vec{a} \times (\vec{b} + \vec{c})$
   g. the rectangular equation of the plane containing A, B, and C (the endpoints of the vectors a, b, and c respectively)
   h. the area of the triangle formed by the points A, B, and C (endpoints of the vectors a, b, and c respectively)

2. Prove—if you can—that $\left(\frac{\vec{u} \cdot \vec{v}}{|u||v|} \right)^2 + \left(\frac{|\vec{u} \times \vec{v}|}{|u||v|} \right)^2 = 1$ Do not use that fact that these are $\sin\theta$ and $\cos\theta$. Let $\vec{u} = u_1 i + u_2 j + u_3 k$ and $\vec{v} = v_1 i + v_2 j + v_3 k$

3. A normal to a plane has direction cosines $\frac{-5}{3\sqrt{5}}, \frac{2}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}$ and $\vec{r} = (3+4t)i + (-1+4t)j + (1+3t)k$ is a line in the plane. Find:
   a. the rectangular equation of the plane.
   b. The distance from the origin to the plane.
   c. The foot of the perpendicular from the origin to the plane.

4. $(2,-3)$ is a point on a line in two-space. The positive unit normal to this line is $\frac{3}{\sqrt{34}} i + bj$ $\quad (b < 0)$. Find the rectangular equation of this line.

5. Find the equations of the lines (in two-space) bisecting the angles formed between the lines $x + 3y = 10$ and $3x + y = 14$.

6. Find the equations of two planes each parallel to the plane $x - 2y + 2z = 6$ and 4 units away from it (the plane).

7. Prove that $-2x + 5y - 3z + 8 = 0$ is perpendicular to $7x + 4y + 2z + 1 = 0$.

8. Find equation(s) for the set(s) of points (x, y, z) that are equidistant from the planes $2x - y - z + 6 = 0$ and $x + 2y - 7z + 12 = 0$. What are these structures?

9. A sphere with center $(7,1,-2)$ is tangent to a plane $2x+3y-z=5$.

   a. What is the radius of the sphere?
   b. What is the vector equation of the line through the center and perpendicular to the plane at the point of tangency?
   c. What are the coordinates of the point of tangency?
   d. What is the point of tangency of the sphere to a parallel plane at the other end of the diameter and what is the rectangular equation of that tangent plane?

10. Find an equation for the plane containing the point $(5,1,-3)$ and perpendicular to each of the following planes: $5x-3y+2z=9$ and $3x+y-4z=17$.
11. Find the volume of the tetrahedron formed with the following vertices: $(1,5,-2),(3,1,4),(0,-1,1),(2,-2,5)$. NB $V=\frac{Bh}{3}$ where B is the area of the base and h is the height.

ANSWERS:

1. a. $\frac{1}{\sqrt{462}}$ b. $\frac{\sqrt{23023}}{154}$ c. $-\frac{6\sqrt{14}}{7}$ d. 55 e. $-5i-15j+5k$ f. $8i+14j+17k$

   g. $32x+y+13z=55$ h. $\frac{\sqrt{1194}}{2}$
2. left to you
3. a. $5x-2y-4z=13$ b. $\frac{13\sqrt{5}}{15}$ c. $\left(\frac{13}{9},-\frac{26}{45},-\frac{52}{45}\right)$
4. $3x-5y=21$
5. $x-y=2$ and $x+y=6$
6. $x-2y+2z=18$ and $x-2y+2z=-6$
7. Show that $\cos\theta=0$ or $\sin\theta=1$ by using dot or cross product (magnitude) formulas.
8. $5x-5y+4z+6=0$ and $7x-y-10z+30=0$ (bisecting planes for the dihedral angles formed by the given planes)
9. a. $\sqrt{14}$ b. $\vec{r}=(7+2t)i+(1+3t)j+(-2-t)k$ c. $(5,-2,-1)$ d. $(9,4,-3)$ and $2x+3y-z=33$
10. $5x+13y+7z=17$
11. $\frac{2}{3}$