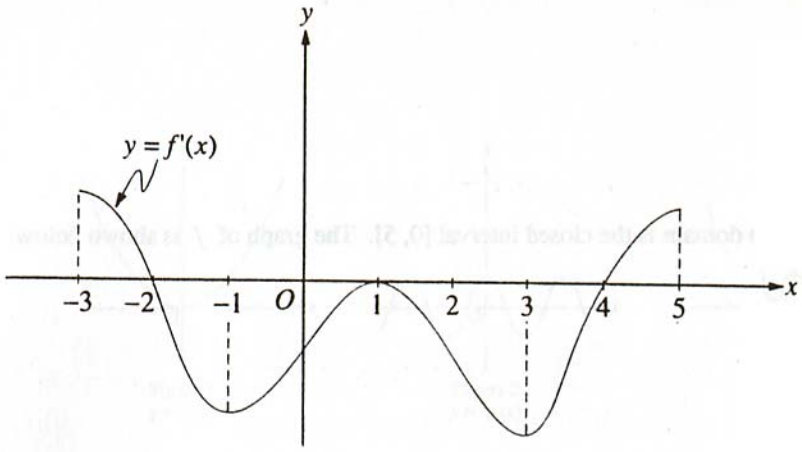
WORKSHEET 4.1,2,3,4,5 (40 points) NAME _____

Please use extra paper for this work. Organize it, show all pertinent steps. Use number lines as appropriate. Use sentences to explain thought processes that don't have computational reasoning. WRITE DARK ENOUGH AND LARGE ENOUGH FOR ME TO READ. Box answers as well as pertinent intermediate results (if you wish). Each part of each problem is worth 2 points.

1. The figure below shows the graph of $f'(x)$, the derivative of a function $f(x)$. The domain of $f(x)$ is the set of all real numbers x such that $-3 < x < 5$.

1996: AB-1



NOTE: This is the graph of the derivative of *f*, NOT the graph of *f*.

   a. For what value(s) of x does $f(x)$ have a relative maximum? Justify your answer.
   b. For what value(s) of x does $f(x)$ have a relative minimum. Justify your answer.
   c. On what interval(s) is the graph of $f(x)$ concave upward? Explain how you can use the given graph of the derivative of $f(x)$ to justify your answer.
   d. Suppose that $f(1) = 0$. Use graph paper and a large enough scale to draw a possible graph of $f(x)$ on the open interval $0 < x < 2$.

2. Let $f(x)$ be the function given by $f(x) = 2\ln(x^2 + 3) - x$ with domain $-3 \le x \le 5$ (2,2,2)

   a. Find the x-coordinate of each relative maximum point and each relative minimum point of $f(x)$. Justify your answer.
   b. Find the x-coordinate of each inflection point of $f(x)$. Justify your answer.
   c. Find the absolute maximum <u>value</u> of $f(x)$.

3. A cubic polynomial function $f(x)$ is defined by $f(x) = 4x^3 + ax^2 + bx + k$ where a, b, and k are constants. The function $f(x)$ has a local minimum at $x = -1$, and the graph of $f(x)$ has a point of inflection at $x = -2$, and $f(2) = 7$. Determine the values of a, b, and k. (This problem is worth 2 points per value)

4. A function $f(x)$ is continuous on the closed interval $[-3,3]$ and $f(-3)=1$ and $f(3)=4$. The functions $f'(x)$ and $f''(x)$ have the properties given in the table below.

x	$-3<x<-1$	-1	$-1<x<1$	1	$1<x<3$
$f'(x)$	Positive	Does not exist	Negative	0	Negative
$f''(x)$	Positive	Does not exist	Positive	0	Negative

   a. What are the *x*-coordinates of all absolute maximum and absolute minimum points of $f(x)$ on the interval $[-3,3]$? State which are max and which are min and justify your answer.
   b. What are the *x*-coordinates of all points of inflection of $f(x)$ on the interval $[-3,3]$? Justify your answer.
   c. On a piece of graph paper, draw a possible graph of $f(x)$ that satisfies all the given properties of $f(x)$.

5. Let $f(x)$ be the function defined for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$ by $f(x)=x+\sin^2 x$
   a. Find EXACT values of *x* for which $f'(x)=1$.
   b. Find EXACT values for the *x*-coordinate(s) of <u>all</u> minimum points of $f(x)$. Justify your answer.
   c. Find EXACT values for the *x*-coordinate(s) of <u>all</u> inflection points of $f(x)$. Justify your answer.

6. Let $h(x)$ be a function defined for all $x \neq 0$ such that $h(4)=-3$ and the derivative of $h(x)$ is given by $h'(x)=\frac{x^2-2}{x}$ for all $x \neq 0$.

   a. Find all values of *x* for which the graph of $h(x)$ has a horizontal tangent, and determine whether $h(x)$ has a local maximum, a local minimum or neither at each of these values. Justify your answers. (3 points for this problem)
   b. On what intervals, if any, is the graph of $h(x)$ concave up? Justify your answer.
   c. Write an equation for the line tangent to the graph of $h(x)$ at $x=4$ (notice that you are given $h(4)$ above).
   d. Does the tangent line to the graph of $h(x)$ at $x=4$ lie above or below the graph for values of $x>4$. Explain.