

All problems are worth 2 points each. Show all appropriate work NEATLY. Do all of your work on separate sheets of paper and box your final answer. When you use graphs or tables or other experimental validation techniques, copy the relevant information (graph, table, etc) and explain in sentences how this supports your answer.

1.
$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 - 16} =$$

2.
$$\lim_{x \rightarrow 5} \frac{\frac{1}{x} - \frac{1}{5}}{3x^2 - 75} =$$

3.
$$\lim_{x \rightarrow e^+} \frac{e - x}{|e - x|} =$$

4.
$$\lim_{x \rightarrow 3} \frac{(7 + 3x)^{1/4}}{(x - 7)^4} =$$

5.
$$\lim_{x \rightarrow \infty} \frac{5x^4 - 3x^3 + x - 4}{8x^5 + 5x^4 - 2x^2 + 5} =$$

6.
$$\lim_{x \rightarrow \infty} mx - \sqrt{m^2 x^2 + px} =$$

7.
$$\lim_{x \rightarrow 0} \frac{5x^4 - 3x^3 + x - 4}{8x^5 + 5x^4 - 2x^2 + 5} =$$

8.
$$\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^3 + 6x^2 + 9x} =$$

9. For $a, b, c \in \text{Reals}$,
$$\lim_{x \rightarrow -\infty} \frac{cx - a}{\sqrt{b^2 x^2 + 1}} =$$

10.
$$\lim_{x \rightarrow 4} \frac{x^2 + 3x}{x - 4} - \frac{6x + 4}{x - 4} =$$

11. Given that $f(x) = 3x^2 - 5x + 1$, find
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

12.
$$\lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{1+h}} - 1}{h} =$$

13.
$$\lim_{x \rightarrow 2} \sqrt{\frac{x^2 - 4}{x^3 - 8}} =$$

14. Find the domain of for x so that $f(x) = \frac{\sqrt{42-3x}}{\sqrt{x-3}}$ is defined (real) and continuous within the interval(s) of the domain. (HINT! Be careful here—this is NOT the same as if everything was under a single radical. Treat the individual parts separately and then combine all of the constraints to give an interval description of the complete domain.
15. Find the domain of $f(x)$ so that $f(x) = \sqrt{(7-5x)(25-4x^2)}$ is defined (real) and continuous within the interval(s) of the domain.
16. Give a NON-TRIVIAL example of a function, $f(x)$, for which $f(x)$ is defined at $x = 5$ and $\lim_{x \rightarrow 5} f(x)$ exists, but for which $f(5) \neq \lim_{x \rightarrow 5} f(x)$. (Hint: find a piecewise definition of a rational function that has these criteria)
17. Determine values for c and d so that $f(x)$ is continuous for all real numbers.
- $$f(x) = \begin{cases} x^2 - 5x + 3 & x < -1 \\ cx + d & -1 \leq x \leq 4 \\ 11 - 3x & x > 4 \end{cases}$$
18. Determine c so that $f(x)$ is continuous for all real numbers.
- $$f(x) = \begin{cases} c^2x^2 - 4cx + 5 & x \leq -2 \\ cx + 11 & x > -2 \end{cases}$$
19. Construct a convincing argument using the Intermediate Value Theorem (but NOT finding an explicit value for x and NOT just drawing a graph) that if $f(x) = x^3 - 5x^2 + 8x - 9$ then there must be a value for x so that $f(x) = 27$.
20. STATE the three-part definition (look it up in your book if you need to!) for continuity and then explain clearly and completely (in sentences) whether $f(x) = \begin{cases} 6x + 4 & x \leq -3 \\ 8 - 2x & x > -3 \end{cases}$ is continuous at $x = -3$. If not, explain explicitly which parts of the definition fail and how.