AB CALCULUS B SEMESTER REVIEW

Show all work on separate sheets

1. Let R be the region in the first quadrant enclosed by the graphs of $f(x)=x^2-2x+2$ and $g(x)=1+2\sin x$.

   a. Use a left-endpoint Riemann sum with four equal subintervals to find an approximation for the area under the curve of $g(x)=1+2\sin x$ on the interval $[0,\pi]$.
   b. Use the trapezoidal rule with five trapezoids to approximate the area under the curve of $f(x)=x^2-2x+2$ on the interval $[0,10]$
   c. Write an expression involving one or more integrals to find the area of the region R.
   d. Write an expression involving one or more integrals to find the volume of the solid generated when R is revolved about the *x*-axis.
   e. The base of a solid is the region R. Each cross section of the solid perpendicular to the *x*-axis is a semicircle. Write an expression involving one or more integrals that gives the volume of the solid.

2. A virulent virus is expected to grow at a rate given by $\frac{dv}{dt}=100\left(3^{0.2t}\right)$ where t is measured in days and v is in hundreds of viral microbes.

   a. Determine the average rate of growth over the first 10 days.
   b. Show the geometric meaning of the average rate of growth on a graph of the rate function and explain its significance.
   c. Find the accumulated numbers of microbes produced during the first 20 days.

3. Evaluate the following integrals:

   a. $\int x\cos\left(3x^2\right)dx$
   b. $\int \frac{x^2}{8+3x^3}dx$
   c. $\int 5e^{\cos 3x}\sin 3x\,dx$
   d. $\int \frac{\sec^2 x}{1+\tan x}dx$
   e. $\int \sin^3 4x\cos 4x\,dx$

4. The acceleration of a particle is given by $a(t)=5+2t$ ft/s^2; $t\ge 0$ At $t=0, v(t)=0$

   a. What is the velocity of the object at $t=3$ seconds?
   b. What is the total distance covered by the object during the first 3 seconds?

5. The graph of a function $f(x)$ is given below. Let $g(x)$ be the function defined by $g(x)=\int_1^x f(t)\,dt$.

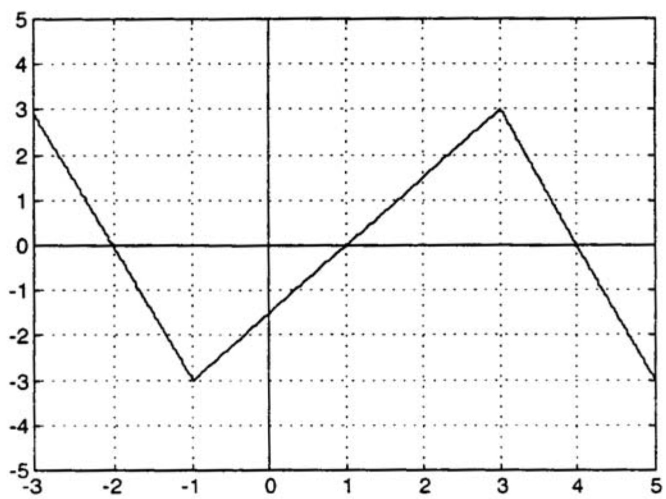
   a. Find each of the following values: $g(1), g(4), g(-1)$

   b. Find all values of x for which $g(x)$ has a relative maximum on the open interval $(-3,5)$. Justify your answer.

   c. Find all values of x for which $g(x)$ is decreasing. Justify your answer.

   d. Write an equation of the tangent line to the graph of $g(x)$ at $x=4$

   e. Find the x-coordinate of each point of inflection of the graph of $g(x)$ on the open interval $(-3,5)$. Justify your answer.



6. Given that $\int_0^5 f(x)\,dx=8$ and that $\int_0^5 g(x)\,dx=4$, evaluate each of the following integrals:

   a. $\int_0^5 \left[f(x)+g(x)\right]dx$

   b. $\int_0^5 \left[3g(x)+x\right]dx$

   c. $\int_0^5 \left[f(x)+4\right]dx$

7. Evaluate each of the following:

   a. $\frac{d}{dx}\int_1^x \frac{dt}{t^2+1}$

   b. If $f(x)=\int_1^x e^t \cos t\,dt$, find $f'(2)$

8. Let $f(x)=3\sin\left(\frac{x}{3}\right)+1$ and $g(x)=x^2-8x+10$

   a. Sketch the graphs of $f(x)$ and $g(x)$ and find their points of intersection.

   b. Find the area enclosed by the graphs of $f(x)$ and $g(x)$

9. Use $u=5x+2$ to write an integral equivalent to $\int_0^3 x^2\sqrt{5x+2}dx$. Be sure to change all occurrences of the variable as well as the limits of integration.

10. A slow-growing bacteria grows at a rate proportional to the amount present where t is measured in days. Set up a differential equation to model this problem and solve it if at $t=0$ there are 20 bacteria present and after 3 days there are 500 bacteria present.

11. Without using a calculator, solve the following integration problems:

   a. $\int_1^e \frac{x+1}{x}dx$

   b. $\int_0^2 \left(3t^2+4t+6\right)dt$

12. Let $f(x)=\frac{\ln x}{x}$. Find the average value of $f(x)$ on $[2,5]$.

13. Solve the following variable separable differential equations with the given initial conditions.

   a. $\frac{dy}{dx}=xy$. Find an expression for y if $y(4)=e$

   b. $\frac{dy}{dx}=\frac{x^2-1}{3y}$. Find an expression for y if $y(0)=2$

14. Let $F(x)$ be an antiderivative of $x\cos\left(x^2\right)$. Find $F(3)$ if $F(0)=0$

15. The rate at which a student learns vocabulary words (where time t is in weeks) is given by the variable separable differential equation $\frac{dw}{dt}=k(5000-w)$. Solve the differential equation if the student already knows 100 of the words at $t=0$ and four weeks later she knows 1000 of them. What is the maximum number of vocabulary words the student can learn? How long will it take until she has mastered all but approximately the last 200 of them?