AB CALCULUS (Slight revision of MCPS review from 2001)
SEMESTER A REVIEW

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Show all work on separate paper

1. Evaluate each of the following limits (show work):

   a. $\lim_{x\to 3}\frac{x^2-9}{x^2-5x+6}$

   b. $\lim_{x\to 0}\frac{5x}{3\sin x}$

   c. $\lim_{x\to \infty}\frac{7-x^3}{2x^3-5x}$

   d. $\lim_{x\to \infty}\frac{5x^3-2x+1}{x-4}$

   e. $\lim_{x\to \infty}\frac{x-9}{3x^4-6x}$

   f. $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}}{h}$

   g. $\lim_{x\to 2}\frac{x^2+2x-8}{x-2}$

   h. $\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x-a}$

2. Which of the following functions is/are continuous at $x=1$?

   $f(x)=\frac{e^{x-1}}{x-1}$ $\quad$ $g(x)=\ln\left(x^2-1\right)$ $\quad$ $h(x)=\frac{\sin x}{x}$

3. Find $\frac{dy}{dx}$ for each of the following functions (show work):

   a. $y=\ln(5x)-\frac{9}{x}$

   b. $y=\frac{3x-4}{x+2}$

   c. $y=\left(3x^2+5\right)\cos x$

   d. $y=\sqrt{x^3+7}$

   e. $y=\sin^2(6x)+x^{4/3}$

   f. $y=f\left[g(x)\right]$

   g. $y=x^2\sin x$

   h. $y=\sin^{-1}(3x)+\tan x-\frac{1}{x^3}$

   i. $y=5\left(2x^3+1\right)+5\text{Arctan}(2x)$

4. Let $g(x)=f\left(x^2+1\right)$. Determine the simplest form of each of the following: (show work)

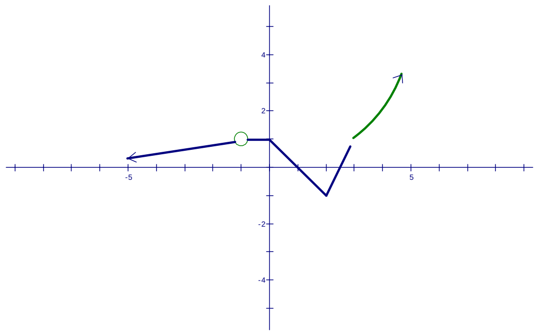
   a. $g'(x)$

   b. $g'(2)$

   c. $g'\left(x^3\right)$

   d. $\frac{d}{dx}\left[g\left(x^3\right)\right]$

5. Use the graph of $f(x)$ below to determine each of the following intervals:



   a. Where is $f(x)$ continuous?

   b. Where is $f(x)$ differentiable?

6. Find the value for *a* that makes $f(x)=\begin{cases} x^2-3 & x<2 \\ ax & x\ge 2 \end{cases}$ continuous at $x=2$. Why can we not NOW ask the question about what value of *a* makes the function differentiable at $x=2$ (Yes, I know you can SOLVE for a value for *a*, but why is this NOT a valid question and/or value?)

7. Suppose that $f(x)$ is continuous on $[-5,5]$ and has the following properties:
   $f(0)=2$, $f(3)=-2$, $f(5)=1$
   $f'' > 0$ on $[\text{-}5,0)$ and $(1.5,5]$, $f'' < 0$ on $[0,1.5]$
   $f(x)$ is decreasing when $x<3$ and
   $f(x)$ is increasing when $x>3$

   Sketch a possible graph of $f(x)$

8. Complete the following problem **WITHOUT** the use of a calculator.
   Given: $f(x)=2x^3-9x^2+12x$
   a. Find the critical value(s) of $f(x)$.
   b. Where is $f(x)$ increasing? Where is $f(x)$ decreasing (use intervals)
   c. Where does $f(x)$ have a relative maximum? Where does $f(x)$ have a relative minimum?
   d. What is the absolute maximum **value** of $f(x)$ on $[0,3]$?
   e. Where does $f(x)$ have inflection point(s)?
   f. Where is $f(x)$ concave up? Where is $f(x)$ concave down?

9. Selected values of a function $h(x)$ where $h''(x)>0$ on $[0,4]$ are given in the table below. Use these values to estimate the value of $h'(3)$.

x	0	1	2	3	4
$h(x)$	.8	1	1.4	2	2.8

10. Given: $f(x)=\ln\left(3x^2+1\right)$

a. Write the linearization (equation of the tangent line) at $x=1$
b. Approximate $f(1.2)$ using your tangent line.
c. Is your approximation larger or smaller than the actual (exact) value. Explain why (not by using a calculator to find the actual value) using calculus.

11. Let $x^2+xy+y^2=27$

a. Find an expression for $\frac{dy}{dx}$.
b. Find the *x*- and *y*- coordinates of all points on the curve where the tangent line is horizontal.
c. Find the *x*- and *y*- coordinates of all points on the curve where the tangent line is vertical.

12. Given $y=4x^3-x^2+5$, find the

a. Average rate of change on $[-1,2]$
b. Instantaneous rate of change at $x=1$

13. Find $\frac{d^2y}{dx^2}$ for each of the following functions (implicitly or explicitly as appropriate).

a. $y=x^3-3\cos(2x)+\ln x$
b. $y=\sqrt{4x-3}$
c. $x^2+y^2=25$

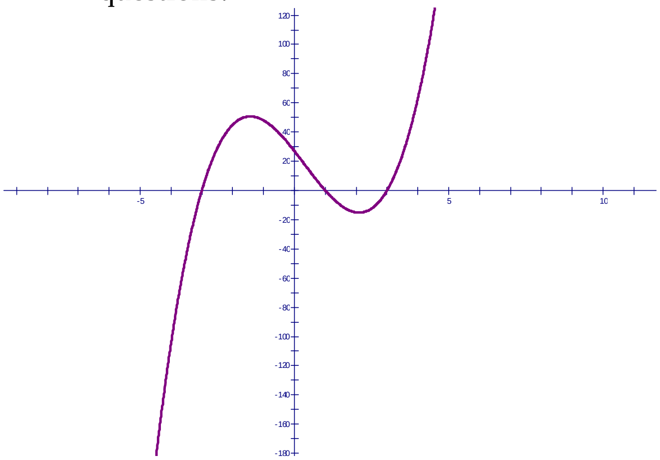
14. The radius of a sphere is increasing at a rate of $2\ \mathrm{ft}/\mathrm{min}$. How fast is the volume changing when the radius is 3 feet? (Recall $V=\frac{4}{3}\pi r^3$)

15. What dimensions would maximize the area of a rectangle with a perimeter of 24 feet?

16. Let $f(x)=3x^3-5x^2+2x+2$

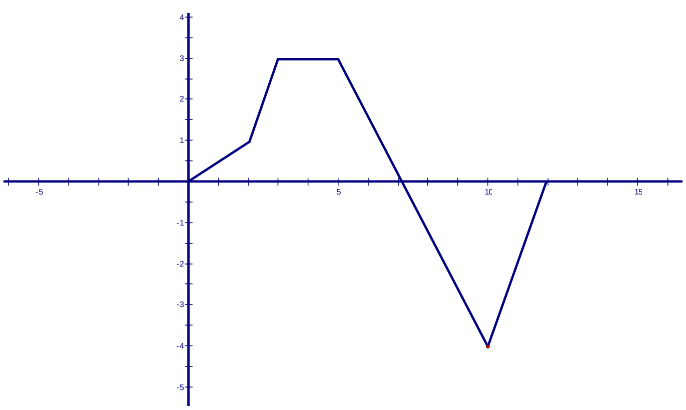
a. Graph this function on $[-1,2]$
b. Use the graph of $f(x)$ to estimate the value(s) of "*c*" guaranteed by the Mean Value Theorem for derivatives on the interval $[-1,2]$.
c. Use calculus to determine the exact actual value(s) of "*c*".

17. Given the graph of $f'(x)$, the **derivative** of $f(x)$ on $[-4,4]$, answer the following questions:



a. On what interval(s) is $f(x)$ increasing? On what interval(s) is $f(x)$ decreasing?
b. Where does $f(x)$ have critical points? How do you know?
c. Where does $f(x)$ have relative maxima? Where does $f(x)$ have relative minima? Justify your answer.
d. Where does $f(x)$ have inflection points? How can you tell?
e. On what interval(s) is $f(x)$ concave up? On what interval(s) is $f(x)$ concave down? Justify your answer.

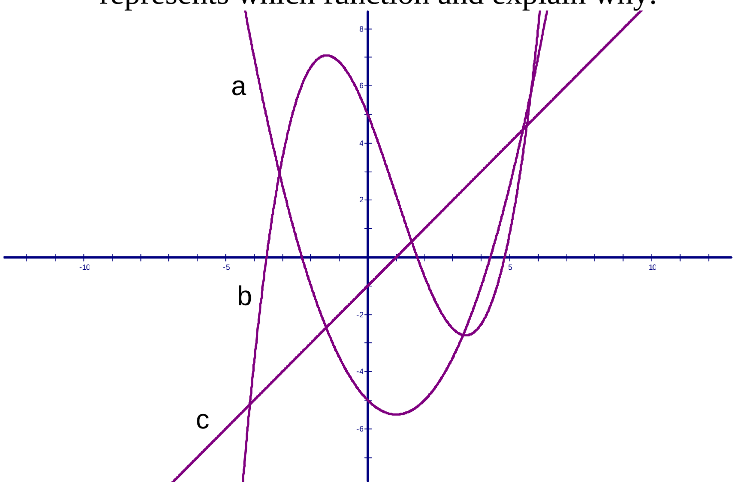
18. The Amazing Fortini is walking along a tightrope 70 feet above the ground. His velocity $v(t)$ at time *t*, $0 \le t \le 12$ is given in the graph below (note that positive velocities mean he is walking to the right, while negative velocities mean he is moving backward—to the left).



a. When does the tightrope walker change direction? Justify your answer.
b. When is his speed (note that speed is an unsigned number value) the greatest? Justify your answer.
c. What is his acceleration at 6 seconds? Justify your answer.

19. On the MCPS review this was once a real question which was morphed at some point into something else that ended up making no sense at all. Because I don't want to redo my own key (and the numbers of the questions on it, there will be NO question in this position.

20. The figure below shows the graphs of $f(x)$, $f'(x)$, and $f''(x)$, Identify which curve represents which function and explain why.



21. Find the equation of the line tangent to the function $y = 3x^2 - 10x + 5$ at the point where $x = 3$.

22. Find the equation of the normal line to the function $y = \cos(5x)$ at the point where $x = \pi$.

23. Given $y = 4x - 3$ find the minimum possible value of the product xy.

24. The function $f(x)$ is continuous and differentiable on $[-5, 5]$ such that $f(-5) = -10$ and $f(5) = 10$. Determine (with explanation which in many cases might be a sketch of a graph that fits the information) which of the following statements are always true, sometimes true, or never true.

a. $f(x)$ is an even function

b. $f(0) = 0$

c. $f'(c) = 0$ for some value of c between -10 and 10

d. $f'(c) > 0$ for all values of x between -5 and 5.

e. $-10 \le f(x) \le 10$ for all values of x between -5 and 5

f. $f'(c) = 0$ for at least one value of c between -5 and 5.

g. $f'(c) = 2$ for at least one value of c between -5 and 5.

h. $f(x) = 9$ for at least one value of x between -5 and 5.

25. Complete the following problem **without the use of a calculator.**

The position of a particle traveling along the *x*-axis is given by $x(t) = t^3 - 6t^2 + 9t + 2$

a. Write an expression for the velocity of the particle at any time *t*.
b. When is the particle at rest? When is it moving left? When is it moving right?
c. What is the speed (not velocity) of the particle at $t = 2$?
d. Write an expression for the acceleration of the particle at any time *t*.
e. What is the minimum velocity of the particle on $[0,3]$?

Wishing each of you much good luck on the exam!