EXPLORATION 1 – MOTIVATING THE "CHAIN RULE"

Before you begin:
Set a friendly window on your calculator [-2,7.4,1,-5,5,1] (82,83) [-1.5,4.8,1,-5,5,1] (85,86) and set TABLE to start at –5 and increment by 1. (Use these throughout this problem set until your graphs go off the top and bottom. Then reevaluate ymin and ymax to accommodate the heights you need.)

Enter the following functions: (for 85,86 all y's will and should be lower case)
Y1=(function to be specified) (83, use a BOLD line to see which graph is which)
Y2=(Y1(x+.0001)-Y1(x-.0001))/.0002 (this is a difference quotient which gives a good numerical approximation of the slope of the tangent—derivative—at each point)

<u>Instructions for the explorations</u>

Enter Y1=sin(x)
Graph (both functions) Realizing that the difference quotient approximates the slope of tangents (derivative), can you hypothesize what function will predict the slope of tangents to the sine function? Enter your hypothesis in Y3. Turn off Y1 and graph Y2 and Y3 (use the BOLD drawing mode for Y3 to watch the path, since if you guess this right you will be redrawing what has already been drawn). Do they seem to coincide? Use TABLE to verify whether they do actually coincide. (These are the steps you should repeat each time you are told to repeat the exploration.)

Clear Y3 for the next few steps (They will mostly go back and repeat the previous instructions. When you are again asked to hypothesize and enter a function into Y3, do so and it will be turned on then.)

Enter Y1=2 sin(x) Repeat the instructions from above.

Now enter Y1=sin(3x) and again repeat the exploration from above. What happens with your hypothesis this time? Do you have any ideas for a way to resolve the discrepancies? To either verify your idea or try to find one, turn off Y1, Y2, and Y3. Enter Y4=Y2/Y3 (the actual derivative values divided by your hypothetical derivative values—this will allow you to see whether you are close but off by some simple value, either something more to be added, subtracted, multiplied or divided through, or perhaps some other rather obvious component that will show up when values are being compared this way). Go to TABLE to investigate these values. Can you form a new hypothesis? Using your hypothesis what do you suppose the derivative of sin(7x) is? (Use the above method to verify this, if you wish)

Now enter $Y1 = \sin(x^2)$ and again repeat the exploration. Looking at the graphs this time you should realize that there is obviously something more going on here. As we move farther to the right (increasing x-values) something is also causing the height of the function to change radically.

Let's try the comparison of the actual derivative (difference quotient) with our hypothesis again to see whether there is some common thread there. Y4=Y2/Y3 Go to TABLE and look at what sorts of numbers you find. Can you update your hypothesis of the derivative. Enter a new Y3 and see whether this one fits better. Why would this happen?