WORKSHEET—CHAPTER 5 (55 points) NAME ______________________________

On extra sheets of paper, show all work associated with the following problems. Do the problems in the order that they appear on the worksheet. Give EXACT answers (not decimal approximations) Box your answers. DO NOT PUT ANSWERS ON THIS PAPER.

1. Find the area of the region under the curve (down to the *x*-axis—this one is a curve that is totally above the *x*-axis so there will not be any ambiguity about what you are to do) $y = 2x^2 - 7x + 10$ between $x = -5$ and $x = 1$. Use right endpoint rectangles (trust me, this is the easier way) and show all steps to take the limit as the number of rectangles approaches infinity (recall how we did this in class). (7 points) (You may wish to refer to the summation formulas on p. 259 as needed)

2. Find exact values for each of the following definite or indefinite integrals. (3 points each)

   a. $\displaystyle\int_9^{25} \frac{5x^2 - 2x + 4}{\sqrt{x}}\,dx$

   b. $\displaystyle\int 7x^2\left(2x^3 - 6\right)^{20} dx$

   c. $\displaystyle\int_2^7 \sqrt{8x - 7}\,dx$

   d. $\displaystyle\int_{-2}^{3} \left(x^4 - 8x^2 - 8x - 9\right) dx$ (Look at the graph and then explain in sentences why this answer comes out negative.)

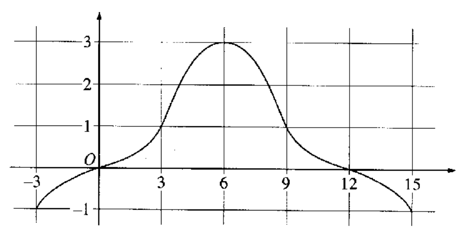
   e. $\displaystyle\int 3\sec x \tan x \sqrt[3]{5 - 2\sec x}\,dx$

   f. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{4\pi}{3}} (3x - \sin 4x)\,dx$

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function $R(t)$ where t is time. The (table below on the left) shows the rate as measured every 3 hours over a 24-hour period. (2 points per part)

   a. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   b. Is there some time t, $0 < t < 24$ such that $R'(t) = 0$? Justify your answer.
   c. The rate of water flow $R(t)$ can be approximated by the function $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

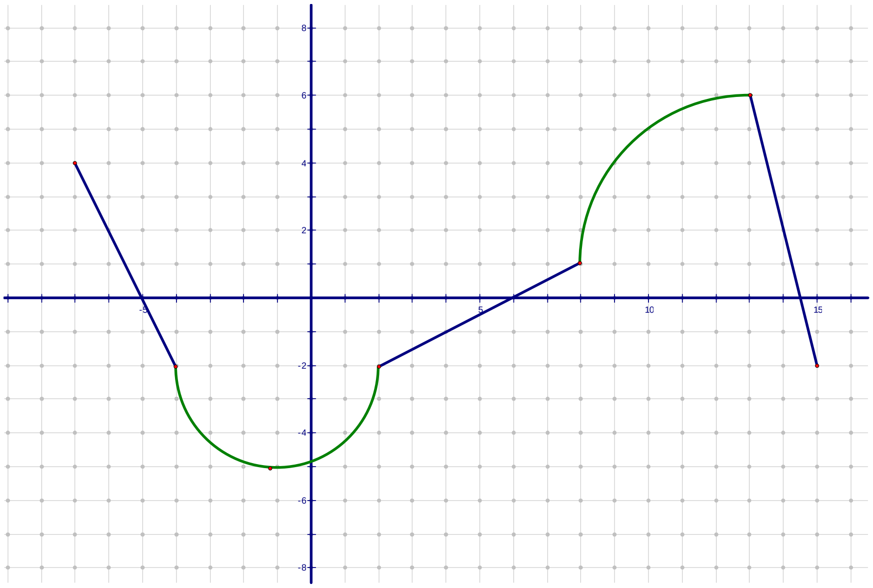


Graph of f

4. The graph of a differentiable function $f(x)$ on the closed interval $[-3,15]$ is shown in the figure above (right). The graph of $f(x)$ has a horizontal tangent line at $x = 6$. Let $g(x) = 2 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$. (2 points per part except for part a, as described below)

   a. Find $g(6), g'(6)$, and $g''(6)$ (1 point each)
   b. On what intervals is $g(x)$ decreasing? Justify your answer.
   c. On what intervals is the graph of $g(x)$ concave down? Justify your answer.
   d. Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six equal subintervals of length 3 (show a drawing and all computations)

5. The graph of a function $f(x)$ (shown below) consists of three straight line segments and one semicircle and one quarter-circle. Evaluate (exact answer) each of the integrals below (1 point each)



a. $\int_{-4}^{8} f(x)\,dx$

b. $\int_{-1}^{2} f(x)\,dx$

c. $\int_{2}^{-4} f(x)\,dx$

d. $\int_{8}^{13} f(x)\,dx$

e. $\int_{-1}^{13} f(x)\,dx$

f. $\int_{2}^{8} f(x)\,dx$

g. $\int_{5}^{5} f(x)\,dx$

h. $\int_{-4}^{4} f(x)\,dx$

i. $\int_{-7}^{15} f(x)\,dx$

j. $\int_{-7}^{15} \left|f(x)\right|dx$

k. $\int_{-7}^{-4} f(x)\,dx$

6. Suppose that $f(x)$ and $g(x)$ are integrable functions and that $\int_{-3}^{5} f(x)\,dx = 13$ and $\int_{-3}^{5} g(x)\,dx = -4$. Use the properties of definite integrals to determine the value of each of the following integrals. (1 point per part)

a. $\int_{-3}^{5} \left(f(x) + g(x) \right) dx = .$

b. $\int_{-3}^{5} \left(f(x) - 2g(x) \right) dx =$

c. $\int_{-3}^{5} \left(g(x) + 4 \right) dx =$

d. $\int_{-3}^{5} \frac{f(x)}{3}\,dx =$