

Using extra paper, show all steps clearly and attach the work. Give EXACT answers and simplify all derivatives at least to the point where like terms have been combined. Box answers. Unless otherwise indicated, each problem or part is worth 2 points.

- If f and g are differentiable functions and $f(4) = 5$, $f'(4) = -3$, $g(4) = 7$, $g'(4) = -2$, find the derivative of $\frac{g}{f}(4)$.
- $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} =$
- If none of $a, b, c, d = 0$, then use the properties of the limits of the trig functions to **show how** to determine the value for $\lim_{x \rightarrow 0} \frac{d \sin(ax)}{c \sin(bx)}$ and use it to find that limit.
- If $f(t) = t^2 \cos t - 2t \sin t$, find $f'\left(\frac{\pi}{2}\right)$
- Using the product rule only, find $f'(x)$ for $f(x) = (3x - 2 \cos x)(5 + 2 \sin x)$ Next multiply the function out and find the derivative again (you may need to use product rule for one or more individual terms here too), simplifying it fully. You should be able to demonstrate that both approaches give the same answer. (2 points per part—4 points total)
- If $f(x) = 8x^7 - 7x^6 + 3x^4 - 2x^3 + 6x - 9$, find $f''''(x)$ (Give your answer in simplest form).
- If $f(t) = \frac{4t^3 - 5t + 2}{3t^2 + 2}$, find $f'(t)$. Simplify the numerator fully and combine like terms.
- If $f(x) = 6x^{\frac{2}{3}} - 10x^{\frac{8}{5}}$, find an equation of the tangent line at the point where $x = -1$.
- Find the coordinates of the points of intersection of $f(x)$ and $f'(x)$ if $f(x) = x^3 + 5x^2 - 7x + 11$. (3 points)
- Without using the chain rule, develop a derivative for $f(x) = \cos(2x)$. (Hint, find a suitable formula for $\cos(2x)$ that lends itself to the rules we are using here--product, quotient and power) (3 points)

11. From the 1984 Advanced Placement Exam: (points 1,2,2). Let f be a function defined by

$$f(x) = \frac{x + \sin x}{\cos x} \text{ for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

- State whether f is an even or an odd function or neither and justify your answer.
- Find $f'(x)$.
- Write an equation of the line tangent to the graph of f at the point where $x = \frac{\pi}{6}$.

12. From the 1978 Advanced Placement Exam: (points 1,2,2) Given the function

$$f(x) = x^4 - x^3 - 11x^2 + 9x + 18$$

- Find the zeroes of f . (Do NOT just use a graph—use an algebraic technique to SHOW that these are the zeroes)
- Write an equation of the line tangent to the graph of f at $x = -2$.
- The point (a, b) is on the graph of f and the line tangent to the graph at (a, b) also passes through the point $(3, -8)$ which is not on the graph of f . Find the values of a and b . An algebraic solution is needed.

13. Adapted from the 1981 Advanced Placement Exam: (points 2,2,2). Let $f(x) = x^4 - 3x^2 + 2$

- Find the exact values of the slopes of the tangents at each of the zeroes of f .
- Write an equation of the line normal (that means, perpendicular to the tangent line) to the graph at the point where $x = 1$.
- Find the exact x -coordinate of each point at which the line tangent to the graph is parallel to the line $y = -2x + 4$.