

Worksheet (due Tuesday, April 14, 2009) – 50 points

This selection of six AP problems (compiled from several recent years) will be scored using their rubrics (9 points each – point values for each part will be given in parentheses). You may collaborate with classmates. Please adhere to the calculator/no-calculator guidelines for the problems. Problems 1 and 3 are from an era of NO calculators (all steps and computations must be clearly shown). Problem 2 requires the use of a calculator essentially for all parts. Problems 4, 5, and 6 are from the non-calculator part of the exam and require all work and steps to be shown.

- Let $f(x)$ be the function given by $f(x) = e^{-x}$ and let $g(x)$ be the function given by $g(x) = kx$, with k being the nonzero constant so that the graph of $f(x)$ is tangent to the graph of $g(x)$. (3 points per part)
 - Find the x -coordinate of the point of tangency and the value of k .
 - Let R be the region enclosed by the y -axis and the graphs of $f(x)$ and $g(x)$. Using the values found above, determine the area of R .
 - Set up an integral and then integrate it to determine the volume of the solid generated by revolving the region R around the x -axis.
- The rate at which people enter an amusement park on a given day is modeled by the function $E(t) = \frac{15600}{t^2 - 24t + 160}$. The rate at which people leave the same amusement park on the same day is modeled by the function $L(t) = \frac{9890}{t^2 - 38t + 370}$. Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park. (3,1,3,2 points by part)
 - How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.
 - The price of admission to the park is \$15.00 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11.00. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
 - Let $H(t) = \int_9^t E(x) - L(x) dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain both the meaning of $H(17)$ and $H'(17)$ in the context of the park.
 - At what time t for $9 \leq t \leq 23$, does this model predict that the number of people in the park is a maximum? Justify your answer.

3. (3,3,3 points) Let $f(x)$ and $g(x)$ be continuous functions with the following properties.

(i) $g(x) = A - f(x)$ where A is a constant.

(ii) $\int_1^2 f(x) dx = \int_2^3 g(x) dx$

(iii) $\int_2^3 f(x) dx = -3A$

a. Find $\int_1^3 f(x) dx$ in terms of A .

b. Find the average value of $g(x)$ in terms of A on the interval $[1, 3]$.

c. Find the value k if $\int_0^1 f(x+1) dx = kA$

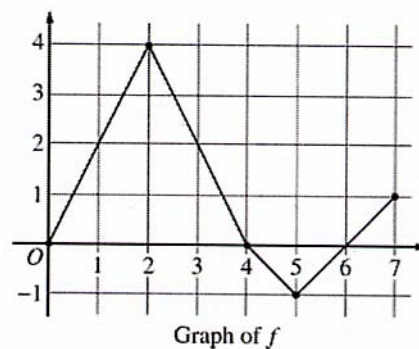
The following problems are to be done without the use of a calculator, so all setups and calculations must be explicitly shown.

4. Let f be a function defined on the closed interval $[0, 7]$.

The graph of f , consisting of four line segments, is shown at the right. Let g be the function given by

$$g(x) = \int_2^x f(t) dt.$$

(3,2,2,2)



a. Find $g(3)$, $g'(3)$, and $g''(3)$.

b. Find the average rate of change of g on the interval $0 \leq x \leq 3$.

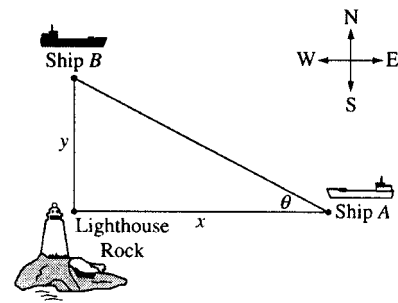
c. Find the absolute minimum value of $g(x)$ on the closed interval $[0, 7]$. Justify your answer.

d. Find the x -coordinate (no y -coordinates are needed or wanted) of each point of inflection on the graph of g on the interval $0 < x < 7$. Justify your answer.

5. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ (I will restate this in a more familiar-looking form: $\frac{dy}{dx} = x\sqrt{y}$) for all real numbers x , where $f(4) = 49$ (3,6)

- Find $f''(4)$.
- Write an expression for $y = f(x)$ by separating the variables and solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(4) = 49$.

6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 km/hr. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t , and let y be the distance between Ship B and Lighthouse Rock at time t , as shown in the figure at the right. (1,4,4)



- Find the distance, in kilometers, between Ship A and Ship B when $x = 30$ km and $y = 16$ km.
- Find the rate of change, in km/hr, of the distance between the two ships when $x = 30$ km and $y = 16$ km.
- Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 30$ km and $y = 16$ km.