NAME _____

Worksheet (due Monday, April 14, 2008) – 50 points

This selection of six AP problems (compiled from several recent years) will be scored using their rubrics (9 points each – point values for each part will be given in parentheses). You may collaborate with classmates. Please adhere to the calculator/no-calculator guidelines for the problems. Problems 1 and 3 are from an era of NO calculators (all steps and computations should be clearly shown). Problem 2 requires the use of a calculator essentially for all parts. Problems 4, 5, and 6 are from the non-calculator part of the exam.

1. Let $f(x)$ be the function given by $f(x) = e^{-x}$ and let $g(x)$ be the function given by $g(x) = kx$, with *k* being the nonzero constant so that the graph of $f(x)$ is tangent to the graph of $g(x)$. (3 points per part)
   a. Find the *x*-coordinate of the point of tangency and the value of *k*..
   b. Let *R* be the region enclosed by the *y*-axis and the graphs of $f(x)$ and $g(x)$. Using the values found above, determine the area of *R*.
   c. Set up an integral and then integrate it to determine the volume of the solid generated by revolving the region *R* around the *x*-axis.

2. The rate at which people enter an amusement park on a given day is modeled by the function $E(t) = \dfrac{15600}{t^2 - 24t + 160}$. The rate at which people leave the same amusement park on the same day is modeled by the function $L(t) = \dfrac{9890}{t^2 - 38t + 370}$. Both $E(t)$ and $L(t)$ are measured in people per hour and time *t* is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the part is open. At time $t = 9$, there are no people in the park. (3,1,3,2 points by part)

   a. How many people have entered the park by 5:00 pm $(t = 17)$? Round your answer to the nearest whole number.
   b. The price of admission to the park is $\$15.00$ until 5:00 pm $(t = 17)$. After 5:00 pm, the price of admission to the park is $\$11.00$. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
   c. Let $H(t) = \int_9^t E(x) - L(x)\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is $3725$. Find the value of $H'(17)$ and explain both the meaning of $H(17)$ and $H'(17)$ in the context of the park.
   d. At what time *t* for $9 \le t \le 23$, does this model predict that the number of people in the park is a maximum? Justify your answer.

3. (3,3,3 points) Let $f(x)$ and $g(x)$ be continuous functions with the following properties.

(i) $g(x) = A - f(x)$ where $A$ is a constant.

(ii) $\int_1^2 f(x)\,dx = \int_2^3 g(x)\,dx$

(iii) $\int_2^3 f(x)\,dx = -3A$

a. Find $\int_1^3 f(x)\,dx$ in terms of $A$.

b. Find the average value of $g(x)$ in terms of $A$ on the interval $[1,3]$.

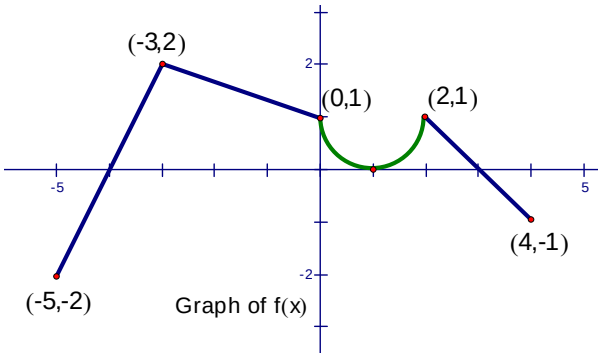
c. Find the value $k$ if $\int_0^1 f(x+1)\,dx = kA$

The following problems are to be done without the use of a calculator, so all setups and calculations must be explicitly shown.

4.

The graph of the function $f(x)$ shown at the right consists of a semicircle and three line segments. Let $g(x)$ be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$

(2,2,3,2 points)




a. Find $g(0)$ and $g'(0)$.

b. Find all values of $x$ in the open interval $(-5,4)$ at which $g(x)$ attains a relative maximum. Justify your answer.

c. Find the absolute minimum value of $g(x)$ on the closed interval $[-5,4]$. Justify your answer.

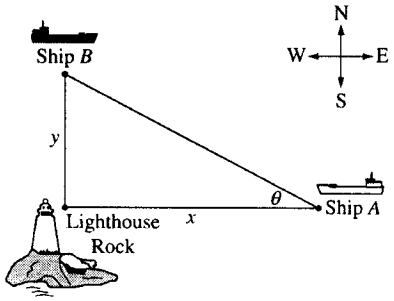
d. Find all values of $x$ in the open interval $(-5,4)$ at which the graph of $g(x)$ has a point of inflection. Justify your answer.

5. Let $f$ be the function satisfying $f'(x) = x\sqrt{f(x)}$ (I will restate this in a more familiar-looking form: $\frac{dy}{dx} = x\sqrt{y}$ ) for all real numbers *x*, where $f(3) = 25$ (3,6)

   a. Find $f''(3)$.

   b. Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$ as given.

6. Ship A is traveling due west toward Lighthouse Rock at a speed of 10 km/hr. Ship B is traveling due north away from Lighthouse Rock at a speed of 15 km/hr. Let *x* be the distance between Ship A and Lighthouse Rock at time *t*, and let *y* be the distance between Ship B and Lighthouse Rock at time *t*, as shown in the figure at the right. (1,4,4)




   a. Find the distance, in kilometers, between Ship A and Ship B when $x = 30$ km and $y = 16$ km.

   b. Find the rate of change, in km/hr, of the distance between the two ships when $x = 30$ km and $y = 16$ km.

   c. Let $\theta$ be the angle shown in the figure. Find the rate of change of $\theta$, in radians per hour, when $x = 30$ km and $y = 16$ km.