# SYNTHETIC DIVISION

In order to divide a polynomial by a first degree polynomial in the form $(x-r)$ we may simplify the process of division considerably by using a method known as "synthetic division".

Eg. $\left(x^4+6x^3-5x^2-12x-4\right)\div(x+2)$ First we show you what long division looks like for this problem.

$$\begin{array}{r|l} & x^3+4x^2-13x+14-\dfrac{32}{x+2} \\ \hline x+2 & x^4+6x^3-5x^2-12x-4 \\ & \underline{x^4+2x^3} \\ & \quad 4x^3-5x^2 \\ & \quad \underline{4x^3+8x^2} \\ & \qquad -13x^2-12x \\ & \qquad \underline{-13x^2-26x} \\ & \qquad\qquad 14x-4 \\ & \qquad\qquad \underline{14x+28} \\ & \qquad\qquad\quad -32 \end{array}$$

Now synthetic division for the same example. But first the instructions:

Put the polynomial in standard form (descending order)

Copy the coefficients with their correct signs but <u>no</u> variables or exponents from the descending order of the polynomial. (In my example below I will put the variables in as column labels but once you get proficient with the method you will not want even those.) Place them in columns corresponding to the descending order of their powers. If any power lacks a term, place a 0 (zero) placeholder in its column as its coefficient. (Since $x^0=1$, the constant goes in the $x^0$ column.

| $x^4$ | $x^3$ | $x^2$ | $x^1$ | $x^0$ |
|---|---|---|---|---|
| +1 | +6 | −5 | −12 | −4 |

Draw a box at the right (Modern books use the left, I have always used the right and I am too old to switch!). Take the divisor binomial, set it equal to zero, and solve for x. Put the value of x in the box. ($x+2=0$ so $x=-2$). Place $-2$ in the box. Skip a line under the line of coefficients and draw a line accross.

$$\begin{array}{rrrrr|l} +1 & +6 & -5 & -12 & -4 & \underline{-2} \\ & & & & & \\ \hline \end{array}$$

Bring down the leading coefficient (far left, corresponding to the highest power). Multiply it by the number in the box. Write this produce above the line but under the second coefficient (ie diagonally above). Find the sum of the two numbers in the column, write it below the line, and repeat the multiplication, entry and summation steps with the answer you just obtained. Repeat this process until you run out of columns (step-by-step expansion is shown on the back of this sheet).

| | | | | | |
|---|---|---|---|---|---|
| +1 | +6 | −5 | −12 | −4 | $\lvert\underline{-2}$ |
| | −2 | | | | |
| 1 | +4 | | | | |

| | | | | | |
|---|---|---|---|---|---|
| +1 | +6 | −5 | −12 | −4 | $\lvert\underline{-2}$ |
| | −2 | −8 | | | |
| 1 | +4 | −13 | | | |

| | | | | | |
|---|---|---|---|---|---|
| +1 | +6 | −5 | −12 | −4 | $\lvert\underline{-2}$ |
| | −2 | −8 | +26 | | |
| 1 | +4 | −13 | +14 | | |

| | | | | | |
|---|---|---|---|---|---|
| +1 | +6 | −5 | −12 | −4 | $\lvert\underline{-2}$ |
| | −2 | −8 | +26 | −28 | |
| 1 | +4 | −13 | +14 | −32 | |

Now look at the "answer" row. Shift each column label one column to the left and match it with the number (coefficient) in the answer row. The polynomial that is constructed here is the quotient. The leftover value at the right is the "remainder" of the division. WAY COOL! Right?

While this method can be proven, let us just comment here that the reason it works is the nice relationship between factors and polynomial values that are underscored by the Factor Theorem and the Remainder Theorem.

Synthetic division can also be quite useful for evaluating a function $f(x)$ for a specific value of $x$ (especially if the value to be used is a number larger than 1 or it is negative, and the degree of the polynomial is higher than about 4)

Eg. $f(x) = 3x^5 - 2x^4 + 7x^2 - 9x + 11$ Find $f(-2)$

Ordinarily this would consist of performing the simplification of the following problem:

$$\begin{aligned} f(-2) &= 3(-2)^5 - 2(-2)^4 + 7(-2)^2 - 9(-2) + 11 \\ &= 3 * -32 - 2 * 16 + 7 * 4 - 9 * -2 + 11 \\ &= -96 - 32 + 28 + 18 + 11 \\ &= -71 \end{aligned}$$

Using synthetic division, it becomes a slightly smoother, less tedious operation. Put the actual value you wish to use for $x$ in the box, and the answer you want is the same as the remainder (This is tantamount to dividing $f(x)$ by $x+2$ except that in this case we are not really interested in knowing what the actual quotient--it's there if we want it--is.).

| | | | | | | |
|---|---|---|---|---|---|---|
| +3 | −2 | 0 | +7 | −9 | +11 | $\lvert\underline{-2}$ |
| ` | −6 | +16 | −32 | +50 | −82 | |
| +3 | −8 | +16 | −25 | +41 | −71 | |