

## MULTIVARIABLE CALCULUS CONCEPTS (SEMESTER 1)

### Chapter 9 – Techniques of Integration

Integration by Parts  
Trig Integration  
Integration using Trig Substitution  
Integration using Partial Fractions  
Integration using Miscellaneous Additional Strategies

### Chapter 11 – Sequences and Series

Determining whether a sequence or series converges  
Deriving a Taylor polynomial (of a finite number of terms) based on given information  
Deriving a Taylor series for a transcendental function by comparing derivative values

### Chapter 14 – Vectors in two and three space

Dot and Cross Products of vectors  
Parametric and vector equations for lines and planes in two and three space  
Rectangular equations for planes in three space  
Volume of a parallelepiped  $\|\vec{a} \times \vec{b} \cdot \vec{c}\|$

Distance from a point to a plane (three space)  $d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$

Distance between skew lines in space  $\frac{|\overline{P_1Q_1} \times \overline{P_2Q_2} \cdot \overline{P_1P_2}|}{\|\overline{P_1Q_1} \times \overline{P_2Q_2}\|}$

Distance from a point to a line (in space)  
Vector (or parametric equations) for line perpendicular to a given line in space  
Vector or parametric equations for the line that is the intersection of two planes

## Chapter 15 – Vector-valued functions

Graphs of paths of vector-valued functions, curves in two and three space

Tangent lines and normal planes to curves

Limits, derivatives, and integrals of vector-valued functions—recalling that these “answers” are themselves VECTOR-VALUED FUNCTIONS!

Motion  $\vec{r}(t)$  = position,  $\vec{r}'(t)$  = velocity,  $\vec{r}''(t)$  = acceleration,  $\|\vec{r}'(t)\|$  = speed

$$\text{Curvature } K = \frac{|y''|}{(1+(y')^2)^{3/2}} = \frac{|f'g'' - g'f''|}{((f')^2 + (g')^2)^{3/2}} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{\|\vec{r}'(t)\|^3} \quad (\text{two and three space})$$

Radius of circle of curvature  $r = \frac{1}{K}$

Finding center of circle of curvature (in two space)

$$\text{Tangential and normal components of acceleration } a_T = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{\|\vec{r}'(t)\|} \quad a_N = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|}$$

## Chapter 16 – Partial Derivatives and their Uses

Multivariable functions  $z = f(x, y)$ ,  $w = f(x, y, z)$

Level curves to surfaces formed by multivariable functions

Limits of multivariable functions

Partial Derivatives

Increments and Differentials and Related Rates

Chain Rules

Tangent planes and normal lines to surfaces

Directional Derivatives  $\nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$  and  $\nabla f \perp$  the surface from which it was computed

Extrema on surfaces (maxima, minima, and saddle points)

Constrained Extrema – solving by setting partials to zero, solving using LaGrange Multipliers