MULTIVARIABLE CALCULUS SEMESTER 1 REVIEW PROBLEMS

1. Find the point of intersection for the line and plane.

   $x = 3 + 8t$

   $y = 4 + 5t \qquad x - 3y + 5z = 12$

   $z = -3 - t$

2. Find equations for the tangent line and normal plane to the curve

   $x = t\cos t \qquad y = 3 + \sin 2t \qquad z = 1 + \cos 3t$ when $t = \frac{\pi}{2}$.

3. Determine whether the two lines intersect. If so, find their point of intersection.

   $l_1: \quad x = 1 + 3t \quad y = 2 - 4t \quad z = 4 - 2t$

   $l_2: \quad x = 5 - v \quad y = -4 + 5v \quad z = 3 + v$

4. Find the angle between a diagonal of a cube and one of its edges (from a common vertex).
5. Find the equation (with integral coefficients throughout) of the plane that passes through $P(-1,2,1)$ and contains the line of intersection of the two planes $x + y - z = 2$ and $2x - y + 3z = 1$.
6. Find the plane which bisects the sphere centered at $C(3,7,4)$ which is parallel to the same sphere's tangent plane at the point $T(7,4,9)$.
7. Find an equation of the plane parallel to the plane $2x - y + 2z + 4 = 0$ if the point $(3,2,-1)$ is equidistant from both planes.
8. Find a point Q with integral coordinates (and then generalize to describe the locations of all such points Q) so that the volume of tetrahedron $ABCQ$ is 25. $A(2,0,-3), B(1,-1,4), C(0,3,1)$ and the volume of a tetrahedron is given by $V = \frac{1}{3}Bh$.
9. Demonstrate that the planes $2x - 3y - z = 5$ and $6x + 3y + 3z = 17$ are perpendicular in two essentially DIFFERENT ways.
10. Find $\vec{r}(t)$ subject to the following conditions: $\vec{r}\,''(t) = 6t\vec{i} - 12t^2\vec{j} + \vec{k}$,

    $\vec{r}\,'(0) = \vec{i} + 2\vec{j} - 3\vec{k}$ AND $\vec{r}(0) = 7\vec{i} + \vec{k}$.

11. Find the length of the parametrized curve for

    $x = e^t\cos t \qquad y = e^t \quad z = e^t\sin t$ for $0 \le t \le 2\pi$.

12. Find the derivative and integral of the vector valued function

    $\vec{r}(t) = \left(2 + 4t^3\right)\vec{i} + \left(e^{-t}\right)\vec{j} + \left(\sin 3t\right)\vec{k}$.

13. A curve has the parametrization $x = 3t^2 + 4 \qquad y = 2t^4 + 3$. Find the curvature and the radius of the circle of curvature at the point $P(16,35)$ for $t > 0$.
14. Find the curvature for the function $y = \dfrac{1}{1+\cos x}$ when $x = \dfrac{\pi}{2}$. Find the coordinates of this point on the curve and the coordinates of the point that is the center of the circle of curvature.
15. Find the limit (or prove that it doesn't exist) $\lim\limits_{(x,y)\to(0,1)} \dfrac{xy - x}{x^2 + y^2 - 2y + 1}$
16. Find the first and second partial derivatives for $f(x, y) = x^4 \ln y + y^2 \cos x + \sin xy + 7$
17. Find the directional derivative of $w = x^2y + y^2z + z^2x$ at $P(1,0,1)$ in the direction of $\langle 0,3,-1\rangle$.
18. Find the directional derivative of $f(x, y) = x^2y - y^2$ at $P(-3,2)$ in the direction of $\vec{a} = 3\vec{i} - 4\vec{j}$.
19. Find the gradient at $P_0(1,-1,3)$ to the surface $x^2 + 2xy - y^2 + z^2 = 7$ and then find the equation of the tangent plane (integral coefficients please).
20. Find an equation for the tangent plane and the normal line to the graph of $xy + 2yz - xz^2 + 10 = 0$ at the point $P(-5,5,1)$
21. Find the tangential and normal components of acceleration for $\vec{r}(t) = t^3\vec{i} + (14t + 1)\vec{j} + 2t\vec{k}$
22. $f(x, y) = xe^{-y} + 3y$ and $P(1,0)$. Find the maximum rate of change of F at the point P, and find the direction in which it occurs.
23. The radius of a right circular cylinder is decreasing at a rate of $1.2\,{}^{cm}\!/_{\sec}$ while its height is increasing at a rate of $3\,{}^{cm}\!/_{\sec}$. At what rate is the volume of the cylinder changing when its radius is $80\,cm$ and its height is $150\,cm$?

24. $z = f(x, y) = \ln\left(\frac{\sqrt{x}}{\sqrt[3]{y}}\right) - e^{xy}$ and $w = g(x, y) = e^{\tan x} \sin y$. Find each of the following partials

a. $f_x(1, 2)$

b. $g_y\left(\frac{\pi}{4}, 0\right)$

c. $\frac{\partial^2 z}{\partial y \partial x}$

d. $\frac{\partial^2 w}{\partial x \partial y}$

e. $g_{xx}(2, 0)$

25. $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx dy$

26. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour the temperature at the point (x, y, z) on the probe's surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.

27. Find the dimensions of the closed curcular can of smallest surface area whose volume is $16\pi \, cm^3$

28. Find three real numbers whose sum is 9 and the sum of whose squares is as small as possible.

29. Find the relative maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

30. $\iint_R x\left(x^2 + y^2\right)^{\frac{1}{2}} dA$ for R bounded by $x^2 + y^2 = 17$ and $y = 0$.

31. Find the volume V of the solid that is bounded by the paraboloid $z = 9 - x^2 - y^2$ and the *xy*-plane.