CONCEPTS FOR THREE-SPACE

Write the general form for a point:

On the *x*-axis On the *y*-axis On the *z*-axis

In the *xy*-coordinate plane In the *xz*-coordinate plane In the *yz*-coordinate plane

On a line through $(5,1,-3)$ and
parallel to the *x*-axis parallel to the *y*-axis parallel to the *z*-axis

In a plane containing $(4,-3,5)$ and parallel to
the *xy*-coordinate plane the *xz*-coordinate plane the *yz*-coordinate plane

On a line through $(1,-4,2)$ and perpendicular to
the *xy*-coordinate plane the *xz*-coordinate plane the *yz*-coordinate plane

Consider these planes (and they are ALL planes in three-space) Where are they? Can you show them on a three-space model? Could you draw a graph of them? Can you describe them in words?

$x = -2$ $y = 5$ $z = 1$

$3x + 2y = 6$ $4y + 3z = 12$ $5x - 3z = 15$

$4x - 2y + 3z = 12$

Traces of a plane are the lines of intersection that the plane makes with the coordinate planes. Identify the traces in each coordinate plane for the planes above. Intercepts of a plane are the points of intersection that the plane makes with the axes. Identify the intercepts for the planes above.

Given two points in space there is just one line through them.

Consider $P_0\left(x_0, y_0, z_0\right)$ and $P_1\left(x_1, y_1, z_1\right)$

$\lambda =$

$\alpha =$ the angle between the positive side of the line and the positive *x*-axis

$\beta =$ the angle between the positive side of the line and the positive *y*-axis

$\gamma =$ the angle between the positive side of the line and the positive *z*-axis

$c_1 = \cos\alpha =$ $c_2 = \cos\beta =$ $c_3 = \cos\gamma =$

Parametric equations for this line (use $P_0\left(x_0, y_0, z_0\right)$ as your starting point)

$x =$

$y =$

$z =$

Equations in the form of $Ax + By + Cz = D$ are planes. The intersection of two non-parallel planes is a line, and the intersection of three non-parallel planes may be a point, although it could also be a line or have no intersection whatsoever.

Consider the following pairs of systems of three planes.

$$\begin{aligned} 19x + 49y - 26z &= 104 \\ 7x + 17y - 8z &= 43 \\ x - 11y + 19z &= 53 \end{aligned}$$

$$\begin{aligned} 10x + 6y - z &= 51 \\ 43x + 57y - 16z &= 153 \\ x + 3y - z &= 9 \end{aligned}$$

Try to solve these systems (as systems of three equations) and you will find that no solution can be found. One system contains three planes that intersect in a single line, the other contains three planes that intersect pairwise in three separate lines. Can you determine which is which?