ANSWERS TO SEMESTER REVIEW

1. Pythagorean Theorem $AB=\sqrt{34}, AC=\sqrt{34}, BC=\sqrt{68}$
2. Outside $d\left((-3,1),(0,4)\right)=\sqrt{18}>4$
3. $1\pm\sqrt{7}$
4. $\dfrac{x+1}{x-2}, x\neq 2$
5. a) $D_f: x\neq 0, D_g: x\neq 1, f\left(g\left(x\right)\right)=x-1, x\neq 1, g\left(f\left(x\right)\right)=\dfrac{x}{1-x}, x\neq 0,1$

   b) $D_f: x\neq -1, D_g: x\neq 0, f\left(g\left(x\right)\right)=x, x\neq 0, g\left(f\left(x\right)\right)=x, x\neq -1$
6. The answer is not unique (choices) $f\left(x\right)=a\left(b\left(x\right)\right)$: $a=2x^2-3, b=1+x$; $a=2x-3, b=\left(1+x\right)^2$
7. $y=\dfrac{x}{1-x}, x<1$
8. Yes
9. First situation could be two short horizontal segments half a unit long, one at $y=k_1$, going across from $x=0$ to $x=\dfrac{1}{2}$ with a closed circle at $x=\dfrac{1}{2}$, the other at $y=k_2$ going across from $x=\dfrac{1}{2}$ (with an open circle at $x=\dfrac{1}{2}$) to $x=1$.

   Second situation would have a vertical asymptote at $x=\dfrac{1}{2}$. Otherwise almost any functional behavior will be acceptable.
10. $f\left(x\right)=\dfrac{2}{3x-5}, f\left(1\right)=-1, f\left(2\right)=2$ but there is a discontinuity at $x=\dfrac{5}{3}$ and there is NO value between 1 and 2 for which $f\left(x\right)=0$.
11. $f\left(x\right)=x^7+3$
12. quotient y^2-y, remainder $y-16$
13. quotient $2x^3-6x^2+17x-48$, remainder 139
14. Remainder $=f\left(-2\right)=0$
15. $f\left(-a\right)=\left(-a\right)^3+3a\left(-a\right)^2+3a^2\left(-a\right)+a^3=-a^3+3a^3-3a^3+a^3=0$. Therefore $x+a$ is a factor.
16. 0
17. $p\left(x\right)=\left(x-1\right)\left(x-2\right)\left(x-3\right)=x^3-6x^2+11x-6$. Other possible answers are any of the form $p\left(x\right)=k\left(x-1\right)^a\left(x-2\right)^b\left(x-3\right)^c$
18. $2, -3$
19. $\left(x-\left(1+2i\right)\right)\left(x-\left(1-2i\right)\right)\left(x-\left(2-i\right)\right)\left(x-\left(2+i\right)\right)=\left(x^2-2x+5\right)\left(x^2-4x+5\right)$

20. $p(x)=(x-2)(x-(3+2i))(x-(3-2i))=(x-2)(x^2-6x+13)=x^3-8x^2+25x-26$

21. a) $x=\frac{2}{3}$ b) $x=-8$ c) $x=\frac{1}{16}$

22. a) horizontal asymptote at $y=-2$, passes through $(0,-1)$ and $(1,0)$, general exponential shape; b) vertical asymptote at $x=-1$, passes through $(0,0)$, $(1,1)$, and $(3,2)$, general log shape

23. $x=6.25=\frac{25}{4}$

24. 4

25. $\frac{15\pi}{2}$

26. $\frac{70\pi}{3}$

27. $\tan\theta=\frac{2}{\sqrt{5}}, \sec\theta=-\frac{3}{\sqrt{5}}$

28. a) sine curve with amp 3, period π, phase shift $\frac{\pi}{2}$ right

b) cosine curve with amp 2, period $\frac{2\pi}{3}$, phase shift $\frac{\pi}{2}$ right

c) tangent curve with period $\frac{\pi}{4}$, phase shift $\frac{3\pi}{4}$ left, asymptotes

$x=-\frac{7\pi}{8}, x=-\frac{5\pi}{8}$

d) cotangent curve with period $\frac{3\pi}{2}$, phase shift $\frac{3\pi}{4}$ right, asymptotes

$x=\frac{3\pi}{4}, x=\frac{9\pi}{4}$

e) cosecant curve with period π, phase shift $\frac{2\pi}{3}$ right, asymptotes

$x=\frac{2\pi}{3}, x=\frac{7\pi}{6}, x=\frac{5\pi}{3}$

f) secant curve with period 4π, phase shift $\frac{5\pi}{3}$ left, asymptotes

$x=-\frac{8\pi}{3}, x=-\frac{2\pi}{3}, x=\frac{4\pi}{3}$

29. 29.4°

30. 233 feet
31. $13.7, 20.6$
32. $75°$, $10.8, 8.8$
33. $38°$
34. 2.2 miles tall
35. 139.6 ft
36. $84.5\,m^2$
37. proof was done in chapter 6 when we studied this stuff
38. a7, b2, c4, d1, e6, f5, g3
39. a) $\frac{56}{65}$ b) $\frac{63}{65}$ c) $-\frac{24}{7}$ d) $\frac{2}{3}$ e) $-\frac{169}{119}$ f) $-\frac{65}{16}$
40. $16i$
41. $-\sqrt{2}+i\sqrt{2}$ and $\sqrt{2}-i\sqrt{2}$
42. $1, -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}-\frac{\sqrt{3}}{2}i$
43. $i, -\frac{\sqrt{3}}{2}-\frac{1}{2}i, \frac{\sqrt{3}}{2}-\frac{1}{2}i$
44. a) $-\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $-\frac{\pi}{3}$ d) $\frac{2\pi}{3}$
45. $x=\frac{1}{2}$
46. a) $\frac{2}{\sqrt{5}}$ b) $\frac{24}{7}$ c) $-\frac{84}{85}$
47. a) $\frac{\pi}{3}, \frac{5\pi}{3}$ $\pm\frac{\pi}{3}+2k\pi=\frac{(6k\pm1)\pi}{3}$ b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ $\frac{\pi}{3}+k\pi, \frac{2\pi}{3}+k\pi$
c) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ $\frac{(12k+1)\pi}{12}, \frac{(12k+5)\pi}{12}$ d) π $(2k+1)\pi$
e) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ $k\pi, \frac{(6k\pm1)\pi}{3}$ f) $\frac{\pi}{2}, \frac{3\pi}{2}$ $\frac{(2k+1)\pi}{2}$
g) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ $\frac{(4k+1)\pi}{2}, \frac{(12k+1)\pi}{6}, \frac{(12k+5)\pi}{6}$
48. $-9x$
49. $m^{4/3}$
50. 36
51. $\sqrt{2}+1+\frac{1}{\sqrt{2}}=\frac{3+\sqrt{2}}{\sqrt{2}}$
52. 3
53. 56
54. $90, 720$