PRECALCULUS WITH ANALYSIS (RL) SEMESTER REVIEW

1. Show that the points $(-3,1),(2,4),$ and $(0,-4)$ are the vertices of a right triangle. What theorem from geometry must you use?
2. Is the point $(0,4)$ inside or outside the circle of radius 4 with center at $(-3,1)$? Draw this figure and explain how you know.
3. Determine y so that $(0,y)$ will be on the circle of radius 4 with center at $(-3,1)$
4. Solve: If $f(x)=x+1$ and $g(x)=x-2$ for all real x, obtain a formula for $f(x)/g(x)$ and sketch the graph. What is the domain of $f(x)/g(x)$?
5. Give domains for $f(x)$ and $g(x)$ and find $f(g(x))$ and $g(f(x))$ and their domains

   a. $f(x)=\frac{1}{x}, \quad g(x)=\frac{1}{x-1}$

   b. $f(x)=\frac{1}{x+1}, \quad g(x)=\frac{1-x}{x}$
6. Decompose the following function into two or more functions each simpler than the given function: $f(x)=2(1+x)^2-3$
7. Find the inverse of $f(x)=\frac{x}{x+1}, \quad x>-1$
8. If a function is continuous on some closed interval will it necessarily be bounded on that interval?
9. Sketch an example of a graph of a function that is defined on $[0,1]$, is bounded, and is continuous everywhere except at $x=\frac{1}{2}$. Sketch an example of another function with domain $[0,1]$, that is unbounded and continuous except at $x=\frac{1}{2}$.
10. Sketch an example of a graph of a function that will illustrate that without the requirement of continuity on an interval, the conclusion of the Intermediate Value Theorem will not be valid. Can you make up a function (state explicitly the expression that represents it) that will do the same?
11. State a polynomial function of degree seven, with leading coefficient 1 and so that $f(0)=3$ (be aware that there are many such functions)
12. Find the quotient and remainder: $(y^4-16)\div(y^2+y+1)$

13. Find the quotient and remainder: $\left(2x^4 - x^2 + 3x - 5\right) \div \left(x + 3\right)$

14. Use the remainder theorem to find the remainder of $\left(x^3 + 3x^2 - 4\right) \div \left(x + 2\right)$

15. Show that $x + a$ is a factor of $x^3 + 3ax^2 + 3a^2x + a^3$.

16. If $p\left(x\right) = 2x^4 - 3x^3 - 12x^2 + 7x + 6$, find $p\left(3\right)$.

17. State a function for $p\left(x\right)$ if the only zeroes are 1, 2, and 3.

18. Find the integral zeroes of $p\left(x\right) = x^4 + x^3 - 5x^2 + x - 6$

19. Two roots of $x^4 - 6x^3 + 18x^2 - 30x + 25 = 0$ are $1 + 2i$ and $2 - i$. Factor the polynomial.

20. State a third degree polynomial with real coefficients that has 2 and $3 + 2i$ as two of its zeroes.

21. Solve for x:

    a. $27^x = 9$

    b. $x^{5/3} = -32$

    c. $x^{3/4} = \frac{1}{8}$

22. Graph:

    a. $y = 2^x - 2$

    b. $f\left(x\right) = \log_2\left(x + 1\right)$

23. Solve for x: $\log x - 4\log 5 = -2$

24. A $60°$ central angle of a circle subtends an arc of length $\frac{4\pi}{3}$ units. What is the radius of the circle?

25. A central angle of a circle with radius 10 has a measure of $\frac{3\pi}{4}$. What is the length of the arc subtended by this angle?

26. A wheel rotates at 700 revolutions per minute (rpm). Through how many radians does a spoke of the wheel rotate in one second?

27. If $\sin\theta = -\frac{2}{3}$ and $W\left(\theta\right) = P$ has P being a point in the third quadrant, what are $\tan\theta$ and $\sec\theta$?

28. Analyze and graph each of the following functions:

a. $y = 3\sin(2x - \pi)$

b. $y = 2\cos\left(3x - \frac{3\pi}{2}\right)$

c. $y = \tan(4x + 3\pi)$

d. $y = \cot\left(\frac{2}{3}x - \frac{\pi}{2}\right)$

e. $y = \csc\left(2x - \frac{4\pi}{3}\right)$

f. $y = \sec\left(\frac{1}{2}x + \frac{5\pi}{6}\right)$

29. What is the angle of elevation of the sun if a 45 foot tall tree casts a shadow 80 feet long?

30. If the angle of depression from the top of a lighthouse 420 feet tall to a boat at sea is $61°$, how far is the boat from the base of the lighthouse?

31. In a parallelogram, the interior angles measure $64°$ and $116°$ and the sides have length 9 and 15. Find the lengths of the diagonals.

32. In a triangle, two angles are $60°$ and $45°$ and the side included between them is 12 units long. Find the measures of the other two sides and of the missing angle.

33. What is the measure of the smallest angle in a triangle with sides 8, 10, and 13 units long?

34. From a point on a level road, the angle of elevation of a mountain peak is found to be $7°$. From a point on the road 4 miles closer to the mountain, the elevation is $9°$. Find the height of the mountain.

35. From the top of a building the angles of depression to two stakes on the ground are $25°$ and $35°$. The stakes are 100 feet apart and in a straight line with the observer. How tall is the building?

36. A triangle has sides of lengths 12.73 inches and 14.86 inches. The included angle has measure $63.35°$. What is the area of the triangle?

37. Show that the area of any triangle is given by $A = \frac{1}{2}ab\sin C$.

38. Match one expression on the left with one expression on the right so as to form a true identity (which you might then prove)

a. $\cos 4x$

b. $\dfrac{1}{1+\cos^2 x}$

c. $1+\tan 2x \tan x$

d. $\tan 2x+\sec 2x$

e. $\tan x+\cot x$

f. $\cos x+\tan x \sin x$

g. $\cot x-\tan x$

1. $\dfrac{\cos x+\sin x}{\cos x-\sin x}$

2. $\dfrac{\sec^2 x}{\tan^2 x+2}$

3. $2\cot 2x$

4. $\sec 2x$

5. $\sec x$

6. $2\csc 2x$

7. $1-8\sin^2 x+8\sin^4 x$

39. Given that $\sin\alpha=\dfrac{12}{13}$ and $\cos\beta=\dfrac{3}{5}$ and α and β are in Quadrant 1, find the following values:

a. $\sin(\alpha+\beta)$

b. $\cos(\alpha-\beta)$

c. $\tan(2\beta)$

d. $\tan\left(\dfrac{1}{2}\alpha\right)$

e. $\sec(2\alpha)$

f. $\csc(\beta-\alpha)$

40. Simplify fully and express in standard (rectangular $a+bi$ form) $\left[2\left(\cos\dfrac{\pi}{8}+i\sin\dfrac{\pi}{8}\right)\right]^4$

41. Find the two square roots of $-4i$.

42. Find the three cube roots of 1.

43. Solve the equation $z^3+i=0$.

44. Evaluate each of the following:

a. $Arc\sin\left(\sin\frac{5\pi}{4}\right)$

b. $Arc\cos\left(\cos\frac{\pi}{3}\right)$

c. $Arc\tan\left(\tan\frac{2\pi}{3}\right)$

d. $Arc\cos\left(\sin\left(-\frac{5\pi}{6}\right)\right)$

45. For what real number(s) x is $Arc\sin\left(2x^2-2x\right)=-\frac{\pi}{6}$?

46. Compute the following values:

a. $\cos\left(\frac{1}{2}Sin^{-1}\frac{4}{5}\right)$

b. $\tan\left[2Cos^{-1}\left(-\frac{3}{5}\right)\right]$

c. $\sin\left[Tan^{-1}\left(-\frac{4}{3}\right)-Cos^{-1}\left(\frac{15}{17}\right)\right]$

47. Solve each of the following for θ with $0\le\theta\le 2\pi$. Try generalizing as well.

a. $4\sin^2\theta+1=8\cos\theta$

b. $3\cot\theta=\tan\theta$

c. $\sin 2\theta=\frac{1}{2}$

d. $\cos\theta+\cos 2\theta+2\sin^2\theta=0$

e. $\sin 2\theta=\sin\theta$

f. $\sin 2\theta=2\cos\theta$

g. $\cos 3\theta\cos 2\theta+\sin 3\theta\sin 2\theta=\sin 2\theta$

48. The 12th term of an arithmetic sequence is $2x$ and the 17th term is $7x$. What is the first term?

49. If the first term of a geometric sequence is $m^{1/3}$ and the third term is $m^{1/2}$, what is the 13th term of the sequence?

50. How many even numbers greater than $40,000$ may be formed by using the digits 3, 4, 5, 6, and 9, if each digit must be used exactly once in each number?

51. The sum of an infinite series is $2\sqrt{2}+2$. If the common ratio is $\frac{1}{\sqrt{2}}$, what is the sum of the first three terms?

52. Find the infinite product: $\left(3^{1/2}\right)\left(3^{1/4}\right)\left(3^{1/8}\right)\left(3^{1/16}\right)\ldots$

53. In how many ways can a committee of four be selected from nine people so as to always include a particular woman (from among the nine)?

54. What is the fifth term in the expansion of $\left(3x+\frac{2}{x}\right)^8$?