Development of the formula for the cosine of the angle between two vectors and definition of the dot product

$$\left\|\overline{AB}\right\|^2 = \left\|\overline{a}\right\|^2 + \left\|\overline{b}\right\|^2 - 2\left\|\overline{a}\right\|\left\|\overline{b}\right\|\cos\theta$$

$$\left((b_1-a_1)^2+(b_2-a_2)^2+(b_3-a_3)^2\right) = \left(a_1^2+a_2^2+a_3^2\right)+\left(b_1^2+b_2^2+b_3^2\right)-2\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}\cos\theta$$

$$b_1^2-2a_1b_1+a_1^2+b_2^2-2a_2b_2+a_2^2+b_3^2-2a_3b_3+a_3^2 = \left(a_1^2+a_2^2+a_3^2\right)+\left(b_1^2+b_2^2+b_3^2\right)-2\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}\cos\theta$$

$$-2a_1b_1-2a_2b_2-2a_3b_3 = -2\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}\cos\theta$$

$$a_1b_1+a_2b_2+a_3b_3 = \sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}\cos\theta$$

$$\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}} = \cos\theta$$

$$\frac{\overline{a}\bullet\overline{b}}{\left\|\overline{a}\right\|\left\|\overline{b}\right\|} = \cos\theta$$

This result suggests the equality of the two parts of the definition of the dot product as well:

$$a_1b_1+a_2b_2+a_3b_3 = \overline{a}\bullet\overline{b} = \left\|\overline{a}\right\|\left\|\overline{b}\right\|\cos\theta$$