

## SUMMARY OF COUNTING RULES

Problems involving counting assortments can be quite difficult. The basic rules are fairly straightforward, but the challenge comes in determining which sort of assortment a problem is describing—how many sets are you choosing from? Are repetitions permitted? Does the order of the elements matter? And many more questions will present themselves. Once these questions are answered, choosing the appropriate strategy may be less difficult.

**Independent choices** – Separate pools of items from which to choose one from each. (Settings include, ordering an automobile, outfits, prix fixe dinners, etc.)

General approach: Form a product each of whose factors is the number of possible items in each pool to be used.

Eg. How many outfits can a person make if s/he has six shirts, two jackets, and five pairs of slacks and if an outfit consists of a shirt, slacks and jacket?

Solution:  $6 \cdot 2 \cdot 5 = 60$

**Selections** – One pool of items from which to select, repeated selections with replacement. Because of the fact that each selection is recorded and then replaced, the pool can never be exhausted, thus selections may consist of as many “items” as we wish with no limitations.

General approach: For a pool containing  $n$  unique items and  $r$  selections, there are  $n^r$  possible selections. Notations  $S(n, r)$  or  ${}_n S_r$

Eg. How many different sets of outcomes can we get when we flip a fair coin twenty times?

Solution:  $2^{20} = 1,048,576$

Eg. How many three-digit numbers can be formed using the digits 1 through 9, inclusive, but no zeroes?

Solution:  $9^3 = 729$

**Permutations** – One pool of items from which items will be chosen (without replacement or repetition). The order in which choices are made is relevant and different orderings of the same elements constitute different permutations.

General approach: There are as many possible choices for the first item to be taken as there are elements in the pool; after an element is selected, there are one fewer elements from which to take the next. The quantity of elements is decreased by one with each choice, and after a certain number of choices (the quantity of elements in the pool) the pool may be exhausted. Notations

and formulas:  $P(n, r)$  or  ${}_n P_r = \frac{n!}{(n-r)!}$  NB. An easier way to compute permutations is to simply

start at  $n$  and construct a product of successively decreasing integer values until you have  $r$  factors.

Eg. How many three-digit numbers can be made with the digits 1 through 9 inclusive if each such number is to contain no number more than once?

Solution:  ${}_9 P_3 = \frac{9!}{6!}$  or, more simply  $9 \cdot 8 \cdot 7 = 504$

**Combinations** – One pool of items from which items will be chosen, however, the order in which they are chosen is irrelevant. The combination will be a group of these items with no ordering associated with them.

General approach: Combinations are formed when the ordering of permutations is no longer relevant. So we can determine the number of permutations for the situation we want and then divide by the number of permutations that any given group of elements could have formed,

ie,  $\frac{P(n,r)}{P(r,r)}$  Notations and formulas:  $C(n,r) = {}_n C_r = \frac{n!}{(n-r)!r!}$  NB. An easier way to compute

combinations is to simply form a fraction in which the numerator is the product of  $r$  factors beginning with  $n$  and counting consecutively downward and the denominator is the product of  $r$  factors beginning with  $r$  and counting consecutively downward (ending with 1).

Eg. How many groups of six objects can you make choosing from a set of nine distinct objects?

$$\text{Solution: } C(9,6) = \frac{9!}{3!6!} \text{ or more simply } \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 84$$

Another concept often associated with selecting items (coins, for example) is the **expected value** or **expectation** of a selection. The expected value is much like an average. It tells the expected outcome with respect to known odds—helpful in evaluating whether a game of chance is worth playing (generally, no!)

General approach: Take the sum of all of the “values” of the possible outcomes and divide it by the total number of possible outcomes (eg. Sum the values of all of the coins and divide by the total number of coins)

Eg., If you are given the chance to draw one coin from a box in which there are 20 pennies, seven nickels, five dimes, and three quarters, but you have to pay twelve cents for the privilege, is this a good game?

Solution:  $E = \frac{20 \cdot 1 + 7 \cdot 5 + 5 \cdot 10 + 3 \cdot 25}{35} = \frac{180}{35} = 5 \frac{1}{7}$  If this is a carnival game, unless they are charging less than your expected value, the game is not worth the risk.