Given two spheres: We subtract one from the other and obtain a plane. In this plane lies a circle that is the circle where the two spheres intersect. The task here is to somehow parametrize a curve that will generate explicitly the points on this circle.

Call the spheres S_1 and S_2 and identify their centers and radii similarly C_1, C_2, r_1, and r_2. Call the center of the circle of intersection C_3 and its radius r_3.

We can find C_3 by writing the vector equation of the line through the two centers of the two spheres. Intersect that line with the plane of intersection and obtain the point C_3.

Likewise we can determine the radius r_3 by using the Pythagorean theorem in the following way. Determine the distance of the center C_1 of one of the spheres from the plane. This is one leg of a right triangle. The desired radius, r_3, is the other leg. The radius, r_1, of the sphere is the hypotenuse.

Determine two perpendicular vectors using the center, C_3, as the starting point and a point in the plane of the circle as the end point. (Is it necessary that the two vectors be perpendicular or would any two vectors in the plane work as well?)

Assuming that we actually do NEED the two vectors to be perpendicular to one another proceed in the following manner: Select ANY point in the plane. The vector between the center of the desired circle and this point is the first vector, $\overline{v_1}$ Compute the cross product between this vector and the vector that is normal to the plane of the circle. This resulting vector, $\overline{v_2}$, will be perpendicular to each of the vectors used in the cross product and thus will be a vector perpendicular to the random vector we have chosen.

Take scalar multiples of these two vectors $\overline{v_1}$ and $\overline{v_2}$ so that the magnitudes of both vectors are equal to the radius of the desired circle, r_3. Now the circle is equal to

$$\bar{r} = \overline{OC_3} + r_3\left(\frac{\overline{v_1}}{\left\|\overline{v_1}\right\|}\cos\theta + \frac{\overline{v_2}}{\left\|\overline{v_2}\right\|}\sin\theta\right).$$