PROPERTIES OF COMPLEX NUMBERS

POWERS OF i: $i=\sqrt{-1}$, $i^2=-1$, $i^3=-i$, $i^4=1$, ...

EQUALITY: Two complex numbers are *equal* if and only if their real parts and their imaginary parts are respectively equal. $a+bi=c+di$ iff $a=c$ and $b=d$

ADDITION: To add two complex numbers, add the real parts to one another and the imaginary parts to one another. (Subtraction can be defined as addition of the opposite, where the opposite of a complex number is merely the complex number in which the real and the imaginary parts are *opposite* in sign from their original expression) $(a+bi)+(c+di)=(a+c)+(b+d)i$

MULTIPLICATION: One can use the FOIL method to multiply two binomials and recall that $i^2=-1$ $(a+bi)(c+di)=(ac)+(ad)i+(bc)i+(bd)i^2=(ac-bd)+(ad+bc)i$

COMPLEX CONJUGATION: The complex conjugate of $a+bi$ is $a-bi$. The importance of the complex conjugate lies in the fact that the product of a complex number and its conjugate is a real number. $(a+bi)(a-bi)=a^2+b^2$

DIVISION: (the complex conjugate is used here to guarantee that the quotient has a rational—in fact real—denominator)

$$a+bi\div c+di=\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

ABSOLUTE VALUE (MODULUS): $|a+bi|$ This value represents the length of the line segment from the origin to the point (a,b) on the complex (Argand) plane and is $|a+bi|=\sqrt{a^2+b^2}$.

ADDITIVE IDENTITY: The complex number that represents the notion of *zero* for addition of complex numbers is $0+0i$. $(a+bi)+(0+0i)=a+bi$

MULTIPLICATIVE IDENTITY: The complex number that represents the notion of *one* for multiplication of complex numbers is $1+0i$. $(a+bi)(1+0i)=a+bi$

ADDITIVE INVERSE: The complex number that when added to $a+bi$ gives the additive identity as a result is $-a-bi$. $(a+bi)+(-a-bi)=(0+0i)$

MULTIPLICATIVE INVERSE: With the exception of the additive identity ($0+0i$), all complex numbers have multiplicative inverses (reciprocals). $\frac{1}{a+bi}=\frac{a-bi}{a^2+b^2}$ (This can be easily derived by simply using the complex conjugate to simplify the original reciprocal expression). NB, as well, that $(a+bi)\cdot\frac{1}{a+bi}=1+0i$